



# HEAT TRANSPORT IN MICRO- AND NANOSCALE THIN FILMS

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A detailed examination of the theory underpinning heat transportation in nanoscale thin films, and explanation of their applications:

- Offers a specialist focus on nanoscale thin films, which will allow the reader to create more efficient heat transfer systems.
- Includes in-depth coverage of the formulation of transient energy transport for short durations of heating, which is valuable to those working in electronics.
- A focus on applications and real-life case studies clearly illustrates how the theories explained in the book can be used in the industry.

*Heat Transport in Micro- and Nanoscale Thin Films* deals with micro-/nanoscale heat transfer in small-scale devices and the thin films. Small-scale parts and thin films are widely used in the electronics industry. However, the drastic change in the thermal conductivity with reducing device size and film thickness modifies the energy transport by heat-carrying phonons in the film. Energy transfer in small-sized devices and thin films deviates from the classical diffusion to radiative transport.

This book presents aspects and applications of the principle methods of heat transport in relation to nanoscale films. These include interface properties of cross-plane transport. It fills the gap between the applications of the physical fundamentals and energy transport at the micro- and nanoscale, which will be valuable for academics, researchers, and students in the fields of materials science and energy transport.

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