Collocation on irregular domains with adaptive hermite element family for transport problems

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Civil Engineering

June, 1989

Abstract

The advection-dispersion transsport equation behaves as a first-order hyperbolic equation rather than as a second-order parabolic equation due to advection dominance. It recognizes immediately the occurrence of a localized behavior or a moving concentration front over the solution domain. A high resolution spatial grid in the vicinity of front is an alternative to solve the advection dominated transport equation. In an attempt to introduce the concept in two dimensions, an extra mid-sided node between the corner nodes of a regular cubic Hermitian element increases the order of the approximation polynomial for that particular side. Obviously, the placement of these extra mid-nodes to that side of the element where the concentration front intersects with the element boundary dictates the formulation of the adaptive element family.

Collocation finite element method is widely known as an efficient numerical approximation technique for solving certain boundary value problems. The method combines the simplicity of collocation with the versatility of deformable finite elements. This is a result of casting the collocation method in a general weighted residual framework and applying coordinate transformations in a way analogous to the finite element procedure. It allows the collocation solution to be defined over general (non-rectangular) domains.

A procedure to formulate the adaptive Hermite element family have been developed by taking products of two appropriate one-dimensional cubic or one-dimensional quintic Hermitian bases, one along the x and other along the y- direction. The occurrence of localized behavior or a moving concentration front over the solution domain necessitate the dynamic placing of these adaptive Hermitian elements in their vicinity. A special technique is devised to condense out the extra mid-sided nodes of the adaptive elements prior to the solution of the global system to keep the matrix banded structure consistent with the elements over the rest of the domain.

Convergence arguments shows that using orthogonal collocation methodology with adaptive Hermite elements have an accuracy O(h8) while the regular cubic Hermite elements O(h4). In regular domains, the numerical solutions are tested against the available on and two-dimensional analytical solutions. A convergence study using an one-dimensional analytical solution is presented. For the discretization attempted, the code was tested up to Peclet No. of 600 in one dimension and 200 in two dimensions. Finally, comparisons are presented with the analytical solutions for a special curvilinear domain and a radial domain. The front tracking scheme worked satisfactorily.