

King Fahd University of Petroleum & Minerals
Department of Mathematical & Statistics

PhD Comprehensive Exam: Algebra Topics

Part 1: Linear Algebra

1-Matrices: Basic properties, Row echelon form, Determinant of a square matrix, Inverse of square matrix, Applications to linear systems.

2-Vector Spaces: Basic properties, Subspaces, Linear independence, Bases, Dimensions of vector spaces, Change of basis, Row space and column space.

3-Linear Transformations: Matrix representations, Kernel and image, Composition and inverse of linear mappings.

4-Inner Products: Basic properties, Orthogonal basis, Orthogonal subspaces, The Gram-Schmidt orthogonalization process.

5- Eigenvalues and Eigenvectors: The Characteristic polynomial, Eigenvalues and eigenvectors, Diagonalization.

6-Linear Functionals: Basic properties, Double duals, Transpose of a linear transformation.

7-Invariant Subspaces: Direct sum decomposition, Primary decomposition.

8-Cyclic subspaces: Cyclic decomposition, Rational form, Jordan form.

9-Linear Operators: Unitary operators, Normal operators.

10-Bilinear forms: Basic properties, Symmetric forms, Skew symmetric forms.

Part 2-Abstract Algebra:

1-Groups: Finite groups and subgroups, Cyclic groups, Permutation groups, Isomorphisms, Cosets and Lagrange's Theorem, External direct products, Normal subgroups and factor groups, Group homomorphisms, Fundamental theorem of finite abelian groups.

2-Rings: rings, Integral domains, Ideals and factor rings, Ring homomorphisms, Polynomial rings, Factorization of polynomials, Divisibility in integral domains.

3-Modules: Basic definitions of modules, Localization, Direct products and sums of modules, Free modules and projective modules, The snake lemma, Tensor products and flatness, Injective modules, Noetherian rings and modules, Hilbert's basis theorem, Nakayama's lemma, Indecomposable modules Semisimplicity, Semisimple rings and structure results.

References:

1- Steven J. Leon, Linear Algebra with Applications, 9th edition, Pearson (2014).

2- J. A. Gallian, Contemporary Abstract Algebra, 8th edition, Cengage Learning (2012).

3- Serge Lang, Linear Algebra, 3rd Edition, Springer (1987).

4- K. Hoffman & R. Kunze, Linear Algebra, 2nd edition, Prentice-Hall (1971).

5- P. Grillet, Abstract Algebra, 2nd edition, Graduate Texts in Mathematics 242, Springer (2007).

6- Serge Lang, Algebra, Revised 3rd Edition, Springer (2002).

Related Courses:

MATH 280, MATH 345, MATH 355, MATH 550, MATH 551.