

King Fahd University of Petroleum & Minerals
Department of Mathematical & Statistics

PhD Comprehensive Exam: Analysis Topics

Part 1: Real Analysis

1. Functions of several real variables:

- Continuous mappings, Differentiable mappings, Partial and directional derivatives, Mean value theorem, Higher order derivatives, Taylor's theorem, Maxima and minima in several variables, Inverse and implicit function theorems.

References

- M. Moskowitz and F. Paliogiannis, Functions of several real variables, World Scientific (2011) [Chapters 2.5- 3.8]
- J. Marsden and M. Hoffman, Elementary classical analysis (1993) [Chapters 5,7,8]

2. Measure Theory and Integration

- Lebesgue Measure: Outer measure, measurable sets, measurable functions, almost everywhere notions, Egoroff's theorem.
- Lebesgue integral: Lebesgue integral of a bounded function, bounded convergence theorem, integral of a nonnegative function, Fatou's lemma, monotone convergence theorem, general Lebesgue integration, dominated convergence theorem.
- General measure and integration: measurable spaces, measurable functions, integration.
- L_p spaces: Minkowski's inequality, Holder's inequality, bounded linear functionals, Riesz representation theorem.

References

- H.L. Royden, Real Analysis, 3rd edition [Chapters 3,4,5,6,7,11-15]
- G. Folland, Real Analysis, Wiley (1999) [Chapters 1,2,3,5,6]

Related Courses:

MATH 411: Advanced Calculus II

Course description: Theory of sequences and series of functions. Continuity and differentiability of functions of several variables. Partial derivatives. The Chain rule. Taylor's theorem. Maxima and minima. Integration of functions of several variables. Convergence and divergence of improper integrals. Derivative of functions defined by improper integrals.

MATH 531: Real Analysis

Course description: Lebesgue measure and outer measure. Measurable functions. The Lebesgue integral. Lebesgue convergence theorem. Differentiation and integration. L_p spaces. Riesz representation theorem. Introduction to Banach and Hilbert spaces

Part 2: Complex Analysis

- Cauchy-Riemann equations, Analyticity
- Harmonic functions, conjugate functions.
- Contour integration.
- Cauchy's Theorem.
- Cauchy integral representation and its consequences.
- Power series.
- Isolated singularities.
- Residues.
- Argument principle, Rouché's Theorem.
- Elementary conformal mappings.

References

- L. Ahlfors, Complex analysis, 3rd edition (1979) [Chapters 2-6.1]
- R. Greene and S. Krantz, Function Theory of One Complex Variable, AMS (2006) [Chapters 1-8]

Related Course:

MATH 533 Complex Variables I

Course description: Analytic functions. Cauchy's theorem and consequences. Singularities and expansion theorems. Maximum modulus principle. Residue theorem and its applications. Compactness and convergence in space of analytic and meromorphic functions. Elementary conformal mappings.