

King Fahd University of Petroleum & Minerals
Department of Mathematical & Statistics

PhD Comprehensive Exam: Differential Equations Topics

Part 1: Partial Differential Equations

- First-order linear, quasilinear and nonlinear PDEs using the method of characteristics: know how to obtain explicit solutions.
- Classification of second-order linear equation: hyperbolic, parabolic and elliptic.
- Separation of variable; eigenfunction expansion.
- The Laplace equation: the fundamental solution; Green's identities; mean value theorem; maximum principle.
- The heat equation: the fundamental solution; maximum principles; Fourier series solutions; energy methods; Gaussian kernel for the pure initial value problem.
- The wave equation: D'Alembert's formula; Kirchhoff's and Poisson's formulas; energy methods; Fourier series solutions; Duhamel's principle; Huygens' principle.

Part 2: Ordinary Differential Equations

- General solutions of first-order equations.
- Second-order linear equations.
- Existence and uniqueness theorems for initial value problems and extendability of solutions to maximal intervals of existence.
- Continuity of solutions; continuous dependence on initial conditions.
- Banach fixed point/contraction mapping theorem.
- Phase plane analysis.
- Linear systems; linearization of nonlinear systems.
- The Poincare-Bendixson theorem; limit sets, attractors.
- Floquet theory.
- Classification of equilibrium solutions of linear and nonlinear systems.
- Energy methods and estimates; Gronwall's inequality.
- Lyapunov functions; stability; nonlinear perturbations.
- Stable/unstable/center manifolds; the Hartman-Grobman theorem.
- Elementary bifurcation theory: one and two dimensional bifurcations.

Suggested references

- Qing Han, A Basic Course in Partial Differential Equations, Graduate Studies in Mathematics, AMS, 2011
- L. Evans, Partial Differential Equations, 2nd Ed., American Mathematical Society, 2010.
- R. McOwen, Partial Differential Equations: Methods & Applications, Prentice-Hall, Inc., 2003.
- Luis Barreira and Claudia Valls, Ordinary Differential Equations: Qualitative Theory, AMS, 2012.
- C. Chicone, Ordinary Differential Equations with applications, 2nd ed., Texts in Applied Mathematics, 2006.

Suggested courses

MATH 568 and MATH 565.