

$$1. \lim_{x \rightarrow -3} \frac{x^2 - 9}{2x^2 + 7x + 3} =$$

Similar to #15/2.3

- (a) $\frac{6}{5}$
 (b) $-\infty$
 (c) 0
 (d) $\frac{1}{2}$
 (e) -3

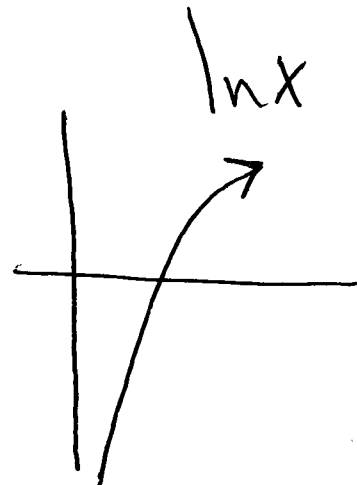
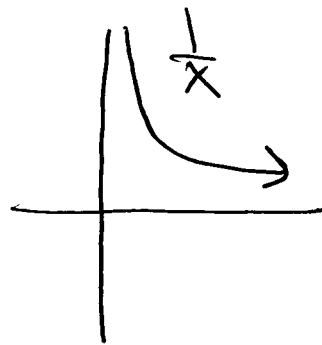
$$\lim_{x \rightarrow -3} \frac{(x-3)(\cancel{x+3})}{(2x+1)(\cancel{x+3})}$$

(correct)

$$= \frac{-6}{-5} = \boxed{\frac{6}{5}}$$

$$2. \lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \ln x \right) =$$

- (a) ∞
 (b) 0
 (c) $-\infty$
 (d) 1
 (e) -1



(correct)

$$\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \ln x \right) = \infty - (-\infty) = +\infty$$

3. The graph of the function $f(x) = \frac{x^3 - 3x^2 + 2x}{x^2 + x}$ has:

- (a) one vertical asymptote
 (b) two vertical asymptotes
 (c) three vertical asymptotes
 (d) no vertical asymptote
 (e) has vertical asymptotes at $x = 0$

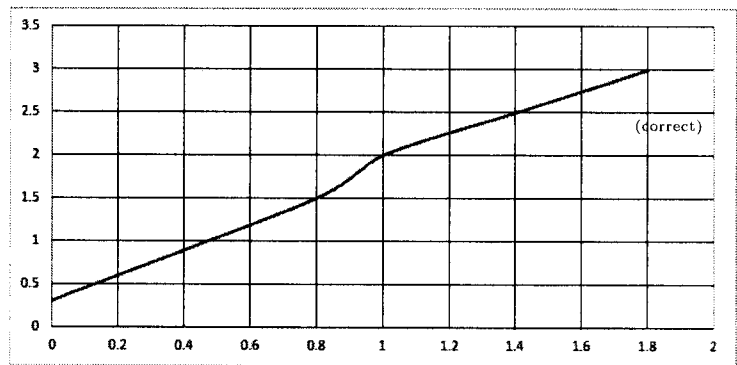
$$\frac{x(x^2 - 3x + 2)}{x(x+1)} \quad (\text{correct})$$

$$\lim_{x \rightarrow -1} \frac{x^2 - 3x + 2}{x + 1} = \infty$$

$$\boxed{x = -1} \quad \lim_{x \rightarrow -1} \frac{(x-1)(x+2)}{(x+1)}$$

4. Use the graph to find δ such that if $0 < |x - 1| < \delta$, then $|f(x) - 2| < 0.5$

- (a) 0.2
 (b) 0.4
 (c) 0.3
 (d) 1
 (e) 2



$$|f(x) - 2| < 0.5 \quad \text{Similar to } \# 2/2.4$$

$$-0.5 < f(x) - 2 < 0.5$$

$$1.5 < f(x) < 2.5 \Rightarrow \text{From graph}$$

$$0.8 \leq x < 1.4$$

$$\text{So } \delta = \min\{1 - 0.8, 1.4 - 1\} = \boxed{0.2}$$

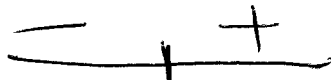
5. $\lim_{x \rightarrow \frac{1}{2}^-} \frac{2x - 1}{|2x^3 - x^2|} =$

Similar to #43/2.3

- (a) -4
- (b) 2
- (c) -2
- (d) 4
- (e) $-\frac{1}{2}$

$\lim_{x \rightarrow \frac{1}{2}^-} \frac{2x - 1}{|x^2| |2x - 1|}$ (correct) $|x^2| = x^2$

$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$



~~$\lim_{x \rightarrow \frac{1}{2}^-} \frac{(2x - 1)}{-x^2(2x - 1)} = \frac{-1}{x^2} = \boxed{-4}$~~

6. $\lim_{x \rightarrow 2} \frac{\sqrt{6-x} - 2}{\sqrt{3-x} - 1} = \frac{2-2}{1-1} = \frac{0}{0}$

- (a) $\frac{1}{2}$
- (b) $-\infty$
- (c) 0
- (d) $\sqrt{2}$
- (e) $-\sqrt{2}$

$\lim_{x \rightarrow 2} \frac{\sqrt{6-x} - 2}{\sqrt{3-x} - 1} \cdot \frac{\sqrt{6-x} + 2}{\sqrt{6-x} + 2} \cdot \frac{\sqrt{3-x} + 1}{\sqrt{3-x} + 1}$ (correct)

$\lim_{x \rightarrow 2} \frac{(\sqrt{6-x})^2 - (2)^2}{(\sqrt{3-x})^2 - (1)^2} \cdot \frac{\sqrt{3-x} + 1}{\sqrt{6-x} + 2}$

$\lim_{x \rightarrow 2} \frac{6-x-4}{3-x-1} \cdot \frac{\sqrt{3-x} + 1}{\sqrt{6-x} + 2}$

$\lim_{x \rightarrow 2} \frac{2-x}{2-x} \cdot \frac{\sqrt{3-x} + 1}{\sqrt{6-x} + 2} = \boxed{\frac{1}{2}}$

Similar to #62 Sec #2.3

7. The horizontal asymptote(s) of $f(x) = \frac{x^2 + x^4}{x^2 - x^4}$ is (are):

- (a) $y = -1$
 (b) $y = 1$
 (c) $y = -1$ and $y = 1$
 (d) $x = 1$
 (e) $x = -1$

$$y = \frac{+1}{-1} = \boxed{-1} \text{ (correct)}$$

Similar to #45/2.6

8. $\lim_{x \rightarrow -\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1} =$

- (a) -3
 (b) 2
 (c) 1
 (d) -1
 (e) 3

$$\frac{\sqrt{x^6 \left(9 - \frac{1}{x^5}\right)}}{x^3 \left(1 + \frac{1}{x^3}\right)}$$

(correct)

$$= \frac{-\cancel{x^3} \sqrt{9 - \frac{1}{x^5}}}{\cancel{x^3} \left(1 + \frac{1}{x^3}\right)}$$

as $x \rightarrow \infty$, $\frac{-3}{1} = \boxed{-3}$

Similar to #23/2.6

9. If $f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$ is continuous everywhere, then $c =$

- (a) $\frac{2}{3}$
 (b) 2
 (c) -1
 (d) $\frac{3}{5}$
 (e) 1

$$\lim_{x \rightarrow 2^-} (cx^2 + 2x) = 4c + 4 \quad (\text{correct})$$

$$\lim_{x \rightarrow 2^+} (x^3 - cx) = 8 - 2c$$

$$4c + 4 = 8 - 2c$$

$$\Rightarrow \boxed{c = \frac{2}{3}}$$

Similar to
#45/2.5

10. Which of the following functions f has a removable discontinuity at a

(I) $f(x) = \frac{x^4 - 1}{x - 1}, a = 1$

(II) $f(x) = \llbracket \sin x \rrbracket, a = \pi$

(III) $f(x) = \frac{1 - x}{1 - |x|}, a = 1$

- (a) (I) and (III) only
 (b) (I) and (II) only
 (c) (I) only
 (d) (I), (II) and (III)
 (e) (II) only

(correct)

$$\text{I): } \lim_{x \rightarrow 1} \frac{(x-1)(x+1)(x^2+1)}{(x-1)} = 4 \quad \checkmark$$

$$\text{II) } \lim_{x \rightarrow \pi^+} \llbracket \sin x \rrbracket = -1, \quad \lim_{x \rightarrow \pi^-} \llbracket \sin x \rrbracket = 0 \quad \times$$

$$\text{III) } \lim_{x \rightarrow 1^+} \frac{1-x}{1-|x|} = \frac{1-x}{1-x} = 1 = \lim_{x \rightarrow 1^-} \frac{1-x}{1-|x|} \quad \checkmark$$

11. Let f be a differentiable function such that $f'(a) = 4$, then $\lim_{h \rightarrow 0} \frac{f(a+5h) - f(a)}{2h} =$

- (a) 10
(b) 5
(c) 2
(d) 11
(e) $\frac{5}{2}$

Re-writes

$$\frac{f(a+5h) - f(a)}{5h} \left(\frac{5}{2}\right)$$

(correct)

let $t = 5h \Rightarrow$

$$\lim_{h \rightarrow 0} \frac{f(a+5h) - f(a)}{5h} = \frac{5}{2} \lim_{t \rightarrow 0} \frac{f(a+t) - f(a)}{t} = \frac{5}{2}(4) = \boxed{10}$$

12. Let $f(x) = ax^2 + bx + 1$. Assume that $f(1) = 2$ and the tangent line to the graph $y = f(x)$ of f at the point $(1, 2)$ has the following equation:

$$y = 3x + 2, \text{ then } 7a + 11b =$$

- (a) 3
(b) 18
(c) 77
(d) 0
(e) 5

$$f(1) = \boxed{a + b + 1 = 2}$$

(correct)

$$f'(1) = \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1}$$

$$\begin{aligned} \text{Now } 2a + b &= 3 \\ a + b + 1 &= 2 \end{aligned}$$

$$\Rightarrow \begin{aligned} a &= 2 \\ b &= -1 \end{aligned}$$

$$\begin{aligned} \text{So } 7(2) + 11(-1) & \\ &= 14 - 11 \\ &= \boxed{3} \end{aligned}$$

$$= \lim_{x \rightarrow 1} \frac{(ax^2 + bx + 1) - (a + b + 1)}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{(ax^2 - a) + (bx - b)}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{a(x^2 - 1) + b(x - 1)}{x - 1}$$

$$\lim_{x \rightarrow 1} a(x+1) + b = \boxed{2a + b = 3}$$

13. Let f be the function defined by

$$\begin{cases} 2x + b, & \text{if } x \leq -2 \\ x^2 + ax + 3b, & \text{if } x > -2 \end{cases}$$

If f is differentiable at $x = -2$, then $4a + 5b =$

$$\begin{aligned} &4(6) + 5(2) \\ &= 34 \end{aligned}$$

- (a) 34
(b) 45
(c) 54
(d) 23
(e) 20

$f(x)$ is cont. at $x = -2$ (correct)

$$f(-2) = \lim_{x \rightarrow -2^-} f(x) = \lim_{x \rightarrow -2^+} f(x)$$

$$-4 + b = 4 - 2a + 3b$$

$$\Rightarrow a - b = 4$$

also, f is Diff. at $x = -2$

$$f'_-(-2) = f'_+(-2) \Rightarrow \text{LHD} = \text{RHD} \Rightarrow 2 = 2(-2) + a$$

$$a = 6$$

$$b = 2$$

14. Which of the following statements is always **True**?

(a) If $\lim_{x \rightarrow a} f(x)$ exists but $\lim_{x \rightarrow a} g(x)$ does not exist, then $\lim_{x \rightarrow a} [f(x) + g(x)]$ does not exist (correct)

(b) If $\lim_{x \rightarrow 5} f(x) = 2$ and $\lim_{x \rightarrow 5} g(x) = 0$, then $\lim_{x \rightarrow 5} \frac{f(x)}{g(x)}$ may exist

(c) If neither $\lim_{x \rightarrow a} f(x)$ nor $\lim_{x \rightarrow a} g(x)$ exists, then $\lim_{x \rightarrow a} [f(x) + g(x)]$ does not exist

(d) If $\lim_{x \rightarrow 3} (f(x)g(x))$ exists, then the limit must be $f(3)g(3)$

(e) If $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} g(x) = \infty$, then $\lim_{x \rightarrow 0} (f(x) - g(x)) = 0$

From T/F Quiz at
End of chapter 2.

15. Let $y = f(x)$ be a function such that

(i) $f(0) = f'(0) = 1$

(ii) $f(2) = f'(2) = 0$

(iii) $f'(1) = -1$

Which of the following graphs could be the graph of $y = f(x)$?

- (a) graph (a)
- (b) graph (b)
- (c) graph (c)
- (d) graph (d)**
- (e) graph (e)

