~ #38
§2.3

1. If $1+\sqrt{x+3} \le g(x) \le x^2-5x-2$, for all x, then $\lim_{x\to 6} g(x) =$

(a) 4

(b) does not exist

(c) 3

(d) 5

(e) 6

By the Squeeze Theorem

(correct)

$\lim_{x\to 6} (1+\sqrt{x+3}) = 1+\sqrt{9} = 1+3 = 4$

$\lim_{x\to 6} (x^2-5x-2) = 36-30-2 = 4$ $\Big] =$

So $\lim_{x\to 6} g(x) = 4$

#32
§2.6

2. $\lim_{x\to\infty} (2\cos(3x)) =$

(a) does not exist

(b) 0

(c) ∞

(d) 2

(e) -2

Since $f(x) = 2\cos(3x)$ oscillates at infinity then the limit does not exist

(correct)

~ #47
§ 2.5

3. If f and g are **continuous** functions such that $g(2) = 4$ and $\lim_{x\to 2}[3f(x) - 2f(x)g(x)] = 20$, then $f(2) =$

$3f(2) - 2f(2)g(2) = 20$

$3f(2) - 2f(2)\cdot 4 = 20$

$-5f(2) = 20$

$f(2) = -4$

f & g are continuum

$\Rightarrow \lim_{x\to 2} f(x) = f(2)$

(correct)

& $\lim_{x\to 2} g(x) = g(2)$

(a) -4

(b) -5

(c) -3

(d) -2

(e) -1

~ #25
§ 2.3

4. $\lim_{t\to 0} \dfrac{\sqrt{1-t^2} - \sqrt{1+t^2}}{t^2} =$

(a) -1

(b) -2

(c) 2

(d) 0

(e) $\dfrac{1}{2}$

$\lim_{t\to 0} \dfrac{\sqrt{1-t^2} - \sqrt{1+t^2}}{t^2} \cdot \dfrac{\sqrt{1-t^2} + \sqrt{1+t^2}}{\sqrt{1-t^2} + \sqrt{1+t^2}}$

(correct)

$\lim_{t\to 0} \dfrac{(1-t^2) - (1+t^2)}{t^2\left(\sqrt{1-t^2} + \sqrt{1+t^2}\right)}$

$\lim_{t\to 0} \dfrac{-2t^2}{t^2\left(\sqrt{1-t^2} + \sqrt{1+t^2}\right)}$

$\lim_{t\to 0} \dfrac{-2}{\sqrt{1-t^2} + \sqrt{1+t^2}} = \dfrac{-2}{1+1} = -1$

~ #49
§2.3

5. $\lim_{x\to -3^-} \frac{x^2-x-12}{|x+3|} =$ $\lim_{x\to -3^-} \frac{(x-4)(x+3)}{-(x+3)}$

$= \lim_{x\to -3^-} -(x-4) = -(-3-4) = 7$

(a) 7 (correct)

(b) -7

(c) 4

(d) -3

(e) 3

~ Example 9
§2.2

6. $\lim_{x\to -2^-} \frac{x-4}{x(x+2)} =$ $\longrightarrow \frac{-6}{0}$, check the sign of $\frac{x-4}{x(x+2)}$ to the left of -2:

$\frac{-}{-(-)} = -$

(a) $-\infty$ (correct)

(b) ∞

(c) 0

(d) -2

(e) 2

$\lim_{x\to -2^-} \frac{x-4}{x(x+2)} = -\infty$

OR $\lim_{x\to -2^-} \frac{x-4}{x} \cdot \frac{1}{x+2} = \frac{-6}{-2} \cdot -\infty = -\infty$

~ #20
§2.5

7. Which one of the following statements is **TRUE** about

$$f(x) = \begin{cases} \dfrac{4-x^2}{2-x} & \text{if} \quad x \neq 2 \\ 1 & \text{if} \quad x = 2 \end{cases}$$

$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{4-x^2}{2-x}$
$= \lim_{x\to 2} (2+x) = 4$ exists
$\neq f(2) = 1$
So f has a removable discontinuity at $x=2$ (correct)

(a) f has a removable discontinuity at $x = 2$

(b) f has a jump discontinuity at $x = 2$

(c) f has an infinite discontinuity at $x = 2$

(d) f is continuous at $x = 2$

(e) f is not defined at $x = 2$

~ #53, 54, 55
§2.3

8. $\lim_{x\to\frac{1}{3}^+} [\![-6x]\!] =$ ($[\![\cdot]\!]$ greatest integer function)

Let $z = -6x$
Since $x \to \frac{1}{3} \Rightarrow z \to -2$
& $x > \frac{1}{3}$ & $-6x < -2$ (correct)
$z < -2$

-2

(a) -1

$\boxed{\text{(b)}}$ -2

(c) does not exist

(d) 2

(e) 3

then
$\lim_{x\to\frac{1}{3}^+} [\![-6x]\!] = \lim_{z\to -2^+} [\![z]\!]$
$= \underline{\underline{-3}}$

~ Example 6, #27 §2.5

9. Where is the function $f(x) = \dfrac{\sqrt{x-1}}{x^3 - 3x^2}$ **continuous**?

(a) $[1,3) \cup (3,\infty)$

(b) $[1,\infty)$

(c) $(0,1) \cup (1,\infty)$

(d) $(-\infty,0) \cup (0,1]$

(e) $(-\infty,0) \cup (0,1] \cup [1,3) \cup (3,\infty)$.

$\sqrt{x-1} \Rightarrow x-1 \geq 0 \Rightarrow x \geq 1$

$x^3 - 3x^2 \neq 0 \Rightarrow x^2(x-3) \neq 0$ (correct)

$\Rightarrow x \neq 0,\ x \neq 3$

0 1 3

$[1,3) \cup (3,\infty)$

~ #45 §2.5

10. If

$$f(x) = \begin{cases} ax^2 + b & \text{if} \quad x < 2 \\ 2bx + a - 3 & \text{if} \quad x \geq 2 \end{cases}$$

is **continuous** at $x = 2$, then $b - a =$

(a) 1

(b) -1

(c) 3

(d) 2

(e) -5

(correct)

f is continuous at $x=2 \Rightarrow \lim_{x\to 2} f(x)$ exists

$\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$\Rightarrow \lim_{x\to 2^-} (ax^2+b) = \lim_{x\to 2^+} (2bx+a-3)$

$\Rightarrow 4a+b = 4b+a-3$

$\Rightarrow 3a-3b = -3$

$\Rightarrow a-b = -1$

$\Rightarrow b-a = 1$

~ #19
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Ch 2 Review

11. $\lim_{x \to 0^+} \tan^{-1}\left(\frac{-1}{x}\right)$

Let $z = -\frac{1}{x}$

if $x \to 0^+$, then $z \to -\infty$

$= \lim_{z \to -\infty} \tan^{-1}(z) = -\frac{\pi}{2}$

(a) $-\frac{\pi}{2}$ (correct)

(b) $\frac{\pi}{2}$

(c) $-\infty$

(d) ∞

(e) 0

~ #20
§ 2.4

12. Let $f(x) = 10 - 7x$ and $\lim_{x \to 2} f(x) = L$. Then the **largest** number δ such that "If $0 < |x - 2| < \delta$, then $|f(x) - L| < 0.1$" is

$L = -4$

$|f(x) - L| < 0.1 \iff |10 - 7x - (-4)| < 0.1$

$\iff |14 - 7x| < 0.1$

$\iff 7|2 - x| < 0.1$

$\iff |2 - x| < \frac{0.1}{7}$

$\iff |x - 2| < \frac{1}{70}$

we may take $\delta = \frac{1}{70}$ (or any smaller value)

$\implies$ the largest value of δ is $\frac{1}{70}$

(a) $\delta = \frac{1}{70}$ (correct)

(b) $\delta = \frac{1}{60}$

(c) $\delta = \frac{1}{80}$

(d) $\delta = \frac{1}{40}$

(e) $\delta = \frac{1}{10}$