

King Fahd University of Petroleum and Minerals  
Department of Mathematics

**Math 101**  
**Major Exam I**  
**213**  
**June 21, 2022**

**EXAM COVER**

**Number of versions: 4**  
**Number of questions: 18**  
**Number of Answers: 5**

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**Net Time Allowed: 120 Minutes**

**MASTER VERSION**

1. Using the definition of the limit for  $\lim_{x \rightarrow 3} \sqrt{x+1} = 2$ , the largest number  $\delta$ , which corresponds to  $\epsilon = 0.1$  is

- (a) 0.39 (correct)  
(b) 0.4  
(c) 0.41  
(d) 0.2  
(e) 0.1

2. Let

$$f(x) = \begin{cases} \frac{6}{(5-x)(4+2x)} & \text{if } x \leq 1 \\ \frac{1}{2\sqrt{x}} & \text{if } x > 1. \end{cases}$$

Then the number of points of discontinuity of  $f$  is

- (a) 2 (correct)  
(b) 0  
(c) 1  
(d) 3  
(e) 4

3. Let

$$f(x) = \begin{cases} \frac{x^2 - 4}{x + 2} & \text{if } x < -2 \\ ax + b & \text{if } -2 \leq x \leq 2 \\ \frac{x^2 - 4}{x - 2} & \text{if } x > 2. \end{cases}$$

If  $f$  is continuous everywhere, then  $a + b =$

- (a) 2
- (b) -2
- (c) 0
- (d) 4
- (e) -4

(correct)

4. The height, in feet, of a ball thrown into the air after  $t$  seconds is given by  $y = 40t - 16t^2$ . The average velocity of the ball on the interval  $[1, 2]$  is

- (a)  $-8 \text{ ft/s}$
- (b)  $8 \text{ ft/s}$
- (c)  $4 \text{ ft/s}$
- (d)  $-4 \text{ ft/s}$
- (e)  $-10 \text{ ft/s}$

(correct)

5. The function  $f(x) = \frac{\sqrt{4x^2 + 3} - x}{x - 1}$  has

- (a) Two horizontal asymptotes  $y = 1$  and  $y = -3$  and one vertical asymptote. (correct)
- (b) Only one horizontal asymptote  $y = 1$  and one vertical asymptote.
- (c) Only one horizontal asymptote  $y = -3$  and one vertical asymptote.
- (d) One vertical asymptote and no horizontal asymptotes.
- (e) No horizontal asymptotes and no vertical asymptote.

6.  $\lim_{x \rightarrow -\infty} (\sqrt{4x^2 + 4x} + 2x) =$

- (a)  $-1$  (correct)
- (b)  $4$
- (c)  $-2$
- (d)  $0$
- (e) DNE

7.  $\lim_{x \rightarrow 2} (\lfloor 2x \rfloor + \lfloor -x \rfloor) =$

- (a) 1
- (b) 6
- (c) 5
- (d) 4
- (e) DNE

(correct)

8.  $\lim_{x \rightarrow 1} (x^2 - 2x + 1) \cos \frac{1}{x-1} =$

- (a) 0
- (b) 1
- (c) 1/2
- (d)  $-\infty$
- (e)  $\infty$

(correct)

9. If  $\lim_{x \rightarrow 2} \frac{10 + x - g(x)}{x - 2} = 3$ , then  $\lim_{x \rightarrow 2} g(x) =$

- (a) 12
- (b) 10
- (c) 3
- (d) 0
- (e) DNE

(correct)

10. Using the Intermediate Value Theorem, we conclude that the two curves  $y = x^3 - x^2 - 1$  and  $y = x^2 - 3$  intersect in the interval

- (a)  $(-1, 0)$
- (b)  $(-2, -1)$
- (c)  $(0, 1)$
- (d)  $(1, 2)$
- (e)  $(2, 3)$

(correct)

11.  $\lim_{h \rightarrow 0} \frac{(2+h)^6 - 64}{h} =$

(a)  $f'(2)$  where  $f(x) = x^6$

(correct)

(b)  $f'(2)$  where  $f(x) = (x+h)^6$

(c) 64

(d) 0

(e) 32

12. Suppose that  $f$  satisfies the equation  $f(x+y) = f(x) + f(y) + x^2y + xy^2$  for all real numbers  $x$  and  $y$ . Suppose also that  $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 1$ . Then  $f'(x) =$

(a)  $1 + x^2$

(correct)

(b)  $x + x^2$

(c)  $x^2$

(d)  $1 + x + x^2$

(e)  $1 + x$



13. If  $f(x) = \frac{1}{\sqrt{x-1}}$ , then  $f'(2) =$

- (a)  $-1/2$
- (b)  $1/2$
- (c)  $1$
- (d)  $2$
- (e)  $-2$

(correct)

14. Consider the function

$$f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ x^3 & \text{if } x < 0 \end{cases}$$

Which of the following is true

- (a) The domain of  $f'(x)$  is  $(-\infty, \infty)$
- (b) The domain of  $f'(x)$  is  $(-\infty, 0) \cup (0, \infty)$
- (c) The domain of  $f''(x)$  is  $(-\infty, \infty)$
- (d) The domain of  $f'''(x)$  is  $(-\infty, \infty)$
- (e)  $f(x)$  is discontinuous at 0

(correct)

15.  $\lim_{x \rightarrow 1} \arcsin \left( \frac{1 - \sqrt{x}}{1 - x} \right) =$

(a)  $\pi/6$

(correct)

(b)  $\pi/3$

(c)  $-\pi/6$

(d)  $-\pi/3$

(e)  $\pi/4$

16. The function  $y = \sqrt[3]{x}$  has

(a) a vertical tangent at  $x = 0$

(correct)

(b) a horizontal tangent at  $x = 0$

(c) a vertical asymptote at  $x = 0$

(d) a horizontal asymptote

(e) a removable discontinuity at  $x = 0$

17. Consider the function  $f(x) = \frac{x^2 - 2x + 1}{x^3 - x}$ .  
Which of the following statements is **FALSE**:

- (a)  $f$  has a removable discontinuity at  $x = -1$  (correct)
- (b)  $f$  has infinite discontinuity at  $x = -1$
- (c)  $f$  has infinite discontinuity at  $x = 0$
- (d)  $f$  has a removable discontinuity at  $x = 1$
- (e)  $f$  has two vertical asymptotes

18. If the equation of the tangent line to the curve  $y = f(x)$  at the point where  $x = 2$  is  $y = 4x - 5$ , then  $f(2) + f'(2) =$

- (a) 7 (correct)
- (b) 6
- (c) 5
- (d) 8
- (e) 3