King Fahd University of Petroleum and Minerals
Department of Mathematics
Math 101
Major Exam II
213
July 25, 2022
Net Time Allowed: 120 Minutes

MASTER VERSION

1. Consider the function

$$f(x) = \begin{cases} x^2 + b & \text{if} \quad x \leq 2 \\ ax + 3 & \text{if} \quad x \geq 2 \end{cases},$$

If f differentiable everywhere, then $a + b =$

(a) 11 (correct)
(b) 8
(c) 15
(d) 10
(e) 13

2. If $g(x) = \dfrac{x}{e^x}$, then $g^{(101)}(0) =$

(a) 101 (correct)
(b) 100
(c) -101
(d) -100
(e) 0

3. If $f(x) = \dfrac{xe^x}{x^2 + e^x}$, then $f'(0) =$

(a) 1 (correct)

(b) e

(c) 0

(d) e^{-1}

(e) e^2

4. The equations of the tangent line(s) to the graph of $f(x) = x^2 - 6x + 9$ that pass through the origin $(0, 0)$ are:

(a) $y = 0$ and $y = -12x$ (correct)

(b) $y = x$ and $y = -12x$

(c) $y = 0$ and $y = 12x$

(d) $y = x$ and $y = 12$

(e) $y = 0$ and $y = x$

5. $\lim\limits_{x\to\frac{\pi}{6}} \dfrac{2\sin x - 1}{x - \dfrac{\pi}{6}} =$

(a) $\sqrt{3}$ (correct)

(b) $\dfrac{1}{2}$

(c) 0

(d) 1

(e) DNE

6. If $f(x) = \dfrac{(\tan x) - 1}{\sec x}$, then $f'\left(\dfrac{\pi}{4}\right) =$

(a) $\sqrt{2}$ (correct)

(b) 1

(c) 0

(d) $-\dfrac{2}{\sqrt{2}}$

(e) $\dfrac{1}{\sqrt{2}}$

7. If $F(x) = f(3f(4f(x)))$, where $f(0) = 0$ and $f'(0) = 2$, then $F'(0) =$

(a) 96 (correct)
(b) 48
(c) 32
(d) 24
(e) 192

8. The slope of the line tangent to the curve $\tan xy = xy^3 + 2y^2 - 8$ at the point $(0, 2)$ is

(a) $-\frac{3}{4}$ (correct)
(b) 2
(c) 0
(d) $\frac{3}{4}$
(e) $\frac{4}{3}$

9. If $5x^2 + 2xy + 2y^2 = 9$, then y'' at the point $(1, 1)$ is

(a) -3 (correct)
(b) 16
(c) 8
(d) -4
(e) 4

10. For any $x > 0$, $\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n$

(a) e^x (correct)
(b) e
(c) e^n
(d) 1
(e) DNE

11. If $y = (\sqrt{x})^x$, then $y'(2) =$

(a) $1 + \ln 2$ (correct)

(b) $2\sqrt{2}$

(c) 1

(d) $1 + \ln(\sqrt{2})$

(e) $\frac{1}{2} + \ln(\sqrt{2})$

12. A particle is moving according to a law of motion $s(t) = \sin\left(\frac{\pi t}{2}\right)$ where t is measured in seconds and s in meters. Then, the total distance, in meters, traveled by the particle during the time interval $0 \leq t \leq 2$ is equal to

(a) 2 (correct)

(b) 1

(c) 0

(d) 3

(e) 4

13. A spotlight on the ground shines on a wall 10 meters away. A woman $2m$ tall walks from the spotlight towards the wall at a speed of $3\,m/s$. When the woman is $4m$ from the building, the length of her shadow on the wall is decreasing at a rate of

(a) $\frac{5}{3}\,m/s$ (correct)

(b) $\frac{30}{8}\,m/s$

(c) $\frac{5}{9}\,m/s$

(d) $\frac{5}{4}\,m/s$

(e) $\frac{10}{9}\,m/s$

14. If we use linear approximation (or differentials) to estimate $(1.009)^9$, then we get $(1.009)^9 \approx$

(a) 1.081 (correct)

(b) 18.1

(c) 1.81

(d) 1.0081

(e) 1.00081

15. The curve $y = x^2 - 2x + \cos(\ln x)$ has a horizontal tangent line at $x =$

(a) 1 (correct)

(b) 0

(c) e

(d) 2

(e) $\dfrac{1}{e}$

16. If $y = x \sin^{-1} x + \sqrt{1 - x^2}$, then $\dfrac{dy}{dx} =$

(a) $\sin^{-1} x$ (correct)

(b) $x \sin^{-1} x$

(c) $\sin^{-1} x + \dfrac{2x}{\sqrt{1 - x^2}}$

(d) $\sin^{-1} x - \dfrac{2x}{\sqrt{1 - x^2}}$

(e) $\dfrac{2x}{\sqrt{1 - x^2}}$

17. Consider the function $y = f(x) = 2^{x^2+1} + \log_2 x$. The rate of change of y with respect to x when $x = 1$ is

(a) $8 \ln 2 + \dfrac{1}{\ln 2}$ (correct)

(b) $64 \ln 2 + \dfrac{1}{\ln 2}$

(c) $1 + 128 \ln 2$

(d) $16 + \dfrac{1}{\ln 2}$

(e) $128 + \dfrac{1}{\ln 2}$

18. A linearization $L(x)$ of the function $f(x) = \sqrt{x} + \sin(x - 1)$ at $a = 1$ is

(a) $L(x) = \dfrac{3}{2}x - \dfrac{1}{2}$ (correct)

(b) $L(x) = \dfrac{3}{2}x + \dfrac{1}{2}$

(c) $L(x) = \dfrac{3}{2}x - \dfrac{3}{2}$

(d) $L(x) = \dfrac{3}{2}x + \dfrac{3}{2}$

(e) $L(x) = \dfrac{1}{2}x - \dfrac{1}{2}$