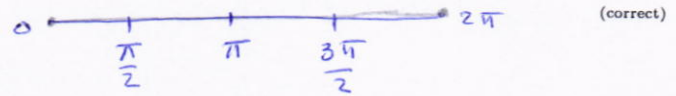


1. Using **four** approximating rectangles and **left endpoints**, the **area** under the graph of $f(x) = \sin\left(\frac{x}{2}\right)$ from $x = 0$ to $x = 2\pi$ is approximately equal to

$$\bullet \Delta x = \frac{2\pi - 0}{4} = \frac{\pi}{2}$$

- (a) $\frac{\pi}{2}(1 + \sqrt{2})$
 (b) $\frac{\pi}{4}(1 + 2\sqrt{2})$
 (c) $\frac{\pi}{4}\left(\frac{1}{2} + \sqrt{2}\right)$
 (d) $\frac{\pi}{4}(2 + \sqrt{2})$
 (e) $\frac{\pi}{3}\left(1 + \frac{\sqrt{2}}{2}\right)$



$$\begin{aligned} A &\approx f(0)\Delta x + f\left(\frac{\pi}{2}\right)\Delta x + f(\pi)\Delta x + f\left(\frac{3\pi}{2}\right)\Delta x \\ &= \Delta x \left[0 + \sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{2}\right) + \sin\left(\frac{3\pi}{4}\right) \right] \\ &= \frac{\pi}{2} \left[0 + \frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2} \right] \\ &= \frac{\pi}{2} (1 + \sqrt{2}) \end{aligned}$$

2. $\int \frac{3t^6 + t^3 + 6}{t^4} dt =$

- (a) $t^3 - \frac{2}{t^3} + \ln|t| + C$
 (b) $t^3 - \frac{2}{t^3} - \frac{1}{t^2} + C$
 (c) $t^3 - \frac{1}{t^3} - \ln|t| + C$
 (d) $2t^3 + \frac{3}{t^3} + \ln|t| + C$
 (e) $t^3 - \frac{1}{t^3} + \ln t + C$

$$\int (3t^2 + \frac{1}{t} + 6t^{-4}) dt$$

$$\begin{aligned} &= t^3 + \ln|t| + 6 \frac{t^{-3}}{-3} + C \\ &= t^3 + \ln|t| - \frac{2}{t^3} + C \\ &= t^3 - \frac{2}{t^3} + \ln|t| + C \end{aligned}$$

(correct)

3. If $f(x) = \begin{cases} 4 & \text{if } -3 \leq x \leq 0 \\ 4 - x^2 & \text{if } 0 < x \leq 3 \end{cases}$, then $\int_{-3}^3 f(x) dx =$

- (a) 15
 (b) 9
 (c) 0
 (d) 12
 (e) 6

$$\begin{aligned} \int_{-3}^3 f(x) dx &= \int_{-3}^0 f(x) dx + \int_0^3 f(x) dx \\ &= \int_{-3}^0 4 dx + \int_0^3 (4 - x^2) dx \\ &= 4x \Big|_{-3}^0 + \left[4x - \frac{x^3}{3} \right]_0^3 \\ &= 4(0 - (-3)) + 4(3) - \frac{3^3}{3} - 0 \\ &= 12 + 12 - 9 \\ &= 15 \end{aligned}$$

(correct)

4. If $G(x) = \int_2^{2x^2} \frac{\sqrt{4+t^2}}{4t} dt$, then $G'(x) =$

- (a) $\frac{\sqrt{1+x^4}}{x}$
 (b) $\frac{\sqrt{1+x^4}}{2x}$
 (c) $\frac{\sqrt{4+4x^4}}{8x^2}$
 (d) $\frac{\sqrt{4+x^4}}{2x^2}$
 (e) $\frac{\sqrt{1+x^4}}{4x}$

$$\begin{aligned} G'(x) &= \frac{\sqrt{4+(2x^2)^2}}{4 \cdot 2x^2} \cdot \frac{d}{dx} [2x^2] \\ &= \frac{\sqrt{4+4x^4}}{8x^2} \cdot 4x \\ &= \frac{\sqrt{4(1+x^4)}}{2x} \\ &= \frac{2\sqrt{1+x^4}}{2x} = \frac{\sqrt{1+x^4}}{x} \end{aligned}$$

(correct)

5. If $\int_{-c}^0 \sqrt{c^2 - x^2} dx = 4\pi^3$, then $c =$

- (a) 4π
 (b) 16π
 (c) 4
 (d) $16\pi^2$
 (e) $4\pi^2$



$$\frac{1}{4} \pi c^2 = 4\pi^3$$

$$\Rightarrow c^2 = 16\pi^2$$

$$\Rightarrow c = 4\pi$$

(correct)

($c > 0$; it follows from the given equation)

6. If $F(x) = \int_{\sqrt{x}}^{x^3} t^2 \cos(\pi t) dt$, then $F'(1) =$

- (a) $-\frac{5}{2}$
 (b) -2
 (c) 0
 (d) $\frac{3}{2}$
 (e) 1

$$\begin{aligned} F'(x) &= (x^3)^2 \cos(\pi x^3) \cdot \frac{d}{dx} [x^3] - (\sqrt{x})^2 \cos(\pi \sqrt{x}) \cdot \frac{d}{dx} [\sqrt{x}] \\ &= x^6 \cos(\pi x^3) \cdot 3x^2 - x \cos(\pi \sqrt{x}) \cdot \frac{1}{2\sqrt{x}} \\ &= 3x^8 \cos(\pi x^3) - \frac{1}{2} \sqrt{x} \cos(\pi \sqrt{x}) \end{aligned}$$

(correct)

$$\begin{aligned} F'(1) &= 3 \cos(\pi) - \frac{1}{2} \cos(\pi) \\ &= -3 + \frac{1}{2} \\ &= -\frac{5}{2} \end{aligned}$$

$$7. \quad \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{5i^4}{n^5} + \frac{2i}{n^2} \right) = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[5 \left(\frac{i}{n} \right)^4 + 2 \left(\frac{i}{n} \right) \right] \cdot \frac{1}{n}$$

(a) 2

(b) 1

(c) $\frac{7}{10}$

(d) 3

(e) $\frac{17}{10}$

$$\bullet \quad X_i^* = a + i \Delta x, \quad \Delta x = \frac{1}{n} = \frac{b-a}{n}, \quad X_i^* = \frac{i}{n}$$

$$\Rightarrow a=0, \quad b-a=1$$

$$\Rightarrow a=0, \quad b=1$$

$$\bullet \quad = \int_0^1 (5x^4 + 2x) dx$$

$$= \left[x^5 + x^2 \right]_0^1 = 1 + 1 - 0 = 2$$

(correct)

$$8. \quad \int_{\frac{\pi}{2}}^{2\pi} \sqrt{1 - \sin^2 t} dt = \int_{\frac{\pi}{2}}^{2\pi} \sqrt{\cos^2 t} dt = \int_{\frac{\pi}{2}}^{2\pi} |\cos t| dt$$

(a) 3

(b) 1

(c) 0

(d) -1

(e) $\frac{3}{2}$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -\cos t dt + \int_{\frac{3\pi}{2}}^{2\pi} \cos t dt$$

$$= -\sin t \Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + \sin t \Big|_{\frac{3\pi}{2}}^{2\pi}$$

$$= -(-1-1) + (0-(-1))$$

$$= 2 + 1$$

$$= 3$$

(correct)

9. $\int_e^{e^4} \frac{\ln x}{x \sqrt{\ln x}} dx =$

Let $u = \ln x$. Then $du = \frac{1}{x} dx$

$x = e \Rightarrow u = \ln e = 1$

$x = e^4 \Rightarrow u = \ln e^4 = 4 \ln e = 4(1) = 4$

(a) $\frac{14}{3}$

(b) $\frac{16}{3}$

(c) $\frac{13}{3}$

(d) $\frac{10}{3}$

(e) $\frac{8}{3}$

$\int_1^4 \frac{u}{\sqrt{u}} du$

$= \int_1^4 \sqrt{u} du = \frac{2}{3} u^{3/2} \Big|_1^4$

$= \frac{2}{3} [4^{3/2} - 1]$

$= \frac{2}{3} [8 - 1] = \frac{2}{3} \cdot 7 = \frac{14}{3}$

(correct)

10. Let f be an **odd** and continuous function on $(-\infty, \infty)$. If $\int_3^0 f(x) dx = 2$ and $\int_3^5 f(x) dx = 9$, then $\int_{-5}^0 f(x) dx =$

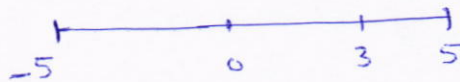
(a) -7

(b) 7

(c) -2

(d) 11

(e) -11



(correct)

Since f is odd, then

$\int_{-5}^5 f(x) dx = 0$

$\Rightarrow \int_{-5}^0 f(x) dx + \int_0^3 f(x) dx + \int_3^5 f(x) dx = 0$

$\Rightarrow \int_{-5}^0 f(x) dx + (-2) + 9 = 0$

$\Rightarrow \int_{-5}^0 f(x) dx = 2 - 9 = -7$