1. The **average value** of $f(x) = x^2 - 6x + 9$ over the interval $[0, 3]$ is equal to

(a) 3

(b) 1

(c) 9

(d) 6

(e) 12

(correct)

$$f_{ave} = \frac{1}{3-0}\int_0^3 (x^2 - 6x + 9)\, dx$$
$$= \frac{1}{3} \cdot \left.\frac{x^3}{3} - 3x^2 + 9x\right]_0^3$$
$$= \frac{1}{3}\left[(9 - 27 + 27) - 0\right]$$
$$= \frac{1}{3} \cdot 9 = 3.$$

2. The **volume** of the solid generated by rotating the region bounded by the curves

$$y = \frac{1}{x^2},\ y = 0,\ x = 1,\ x = 5$$

about the y-axis is equal to

(a) $2\pi \ln 5$

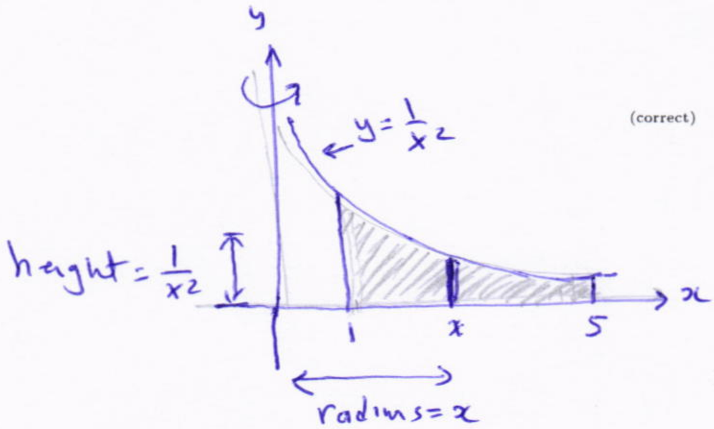
(b) $2\pi \ln 4$

(c) $2\pi \ln 3$

(d) $\pi \ln 5$

(e) $\pi \ln 4$

(correct)



Shell Method

$$V = 2\pi \int_1^5 x \cdot \frac{1}{x^2}\, dx$$
$$= 2\pi \int_1^5 \frac{1}{x}\, dx$$
$$= 2\pi \left.\ln|x|\right]_1^5$$
$$= 2\pi \ln 5.$$

3. $\int 9\sqrt{x}\ln x\,dx =$ $9\int \sqrt{x}\ln x\,dx = 9\int \underbrace{\ln x}_{u} \cdot \underbrace{\sqrt{x}\,dx}_{dv}$

$u = \ln x \quad dv = \sqrt{x}\,dx$

$du = \frac{1}{x}dx \quad v = \frac{2}{3}x^{3/2}$

(a) $6x^{3/2}\ln x - 4x^{3/2} + C$ (correct)

(b) $9x^{3/2}\ln x + 2x^{3/2} + C$

(c) $2x^{3/2}\ln x - 6x^{3/2} + C$

(d) $-6x^{3/2}\ln x + 4x^{3/2} + C$

(e) $6x^{3/2}\ln x - 6x^{3/2} + C$

$= 9\left[uv - \int v\,du\right]$

$= 9\left[\frac{2}{3}x^{3/2}\ln x - \int \frac{2}{3}x^{1/2}\,dx\right.$

$= 9\left[\frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\cdot\frac{2}{3}x^{3/2}\right] + C$

$= 6x^{3/2}\ln x - 4x^{3/2} + C$

4. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^4 t\cdot\sec^4 t\,dt =$ $\int_{-\frac{\pi}{4}}^{\pi/4} \tan^4 t \cdot \sec^2 t \cdot \sec^2 t\,dt$

$= \int_{-\frac{\pi}{4}}^{\pi/4} \tan^4 t\cdot(1+\tan^2 t)\cdot\sec^2 t\,dt$

$u = \tan t \Rightarrow du = \sec^2 t\,dt$ (correct)

$t = -\frac{\pi}{4} \Rightarrow u = -1$

$t = \frac{\pi}{4} \Rightarrow u = 1$

(a) $\frac{24}{35}$

(b) $\frac{8}{15}$

(c) $\frac{16}{35}$

(d) $\frac{4}{35}$

(e) $\frac{1}{5}$

$= \int_{-1}^{1} u^4(1+u^2)\,du$

$= \int_{-1}^{1}(u^4+u^6)\,du = \left.\frac{u^5}{5} + \frac{u^7}{7}\right]_{-1}^{1}$

$= \left(\frac{1}{5}+\frac{1}{7}\right) - \left(-\frac{1}{5}-\frac{1}{7}\right) = \frac{2}{5}+\frac{2}{7} = \frac{14+10}{35} = \frac{24}{35}$

5. $\displaystyle\int_0^{\frac{\pi}{20}} \sin^2(5x)\,dx =$ $\displaystyle \frac{1}{2}\int_0^{\pi/20} [1 - \cos(10x)]\,dx$

$= \frac{1}{2}\left[x - \frac{1}{10}\sin(10x)\right]_0^{\pi/20}$

$= \frac{1}{2}\left[\left(\frac{\pi}{20} - \frac{1}{10}\sin\left(\frac{\pi}{2}\right)\right) - 0\right]$

$= \frac{1}{2}\left(\frac{\pi}{20} - \frac{1}{10}\right)$

$= \frac{1}{40}(\pi - 2)$

(a) $\dfrac{1}{40}(\pi - 2)$ (correct)

(b) $\dfrac{\pi}{40}$

(c) $\dfrac{1}{20}(\pi - 1)$

(d) $\dfrac{1}{20}(2\pi - 1)$

(e) $\dfrac{1}{40}(\pi - 1)$

6. $\displaystyle\int \frac{1}{\sqrt{9x^2+1}}\,dx =$ $\displaystyle\int \frac{1}{\sqrt{(3x)^2+1}}\,dx$. Let $3x = \tan\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

Then $3\,dx = \sec^2\theta\,d\theta$

$\sqrt{(3x)^2+1} = \sqrt{\tan^2\theta + 1} = \sqrt{\sec^2\theta}$

$= |\sec\theta| = \sec\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

(a) $\dfrac{1}{3}\ln|\sqrt{9x^2+1} + 3x| + C$ (correct)

(b) $\dfrac{1}{3}\ln|\sqrt{9x^2+1} + x| + C$

(c) $\ln|\sqrt{9x^2+1} - 3x| + C$

(d) $\ln|\sqrt{9x^2+1} + 6x| + C$

(e) $\dfrac{1}{3}\ln|\sqrt{9x^2+1} - 2x| + C$

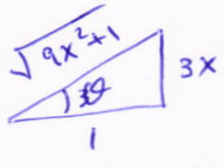
$= \displaystyle\int \frac{1}{\sec\theta}\cdot\frac{1}{3}\sec^2\theta\,d\theta$

$= \frac{1}{3}\displaystyle\int \sec\theta\,d\theta$

$= \frac{1}{3}\ln|\sec\theta + \tan\theta| + C$

$= \frac{1}{3}\ln|\sqrt{9x^2+1} + 3x| + C$

$\tan\theta = 3x$



$\sec\theta = \sqrt{9x^2+1}$

7. The improper integral $\int_0^5 \frac{1}{\sqrt[3]{5-x}}\,dx$ is

(a) convergent and its value is $\frac{3}{2}\sqrt[3]{25}$ (correct)

(b) convergent and its value is $\frac{3}{2}\sqrt[3]{5}$

(c) convergent and its value is $-\frac{3}{2}\sqrt[3]{25}$

(d) convergent and its value is $\frac{3}{4}\sqrt[3]{5}$

(e) divergent

$$= \lim_{t \to 5^-} \int_0^t (5-x)^{-1/3}\,dx$$

$$= \lim_{t \to 5^-} -\frac{3}{2}(5-x)^{2/3}\Big]_0^t$$

$$= \lim_{t \to 5^-} -\frac{3}{2}(t-5)^{2/3} + \frac{3}{2}(5-0)^{2/3}$$

$$= 0 + \frac{3}{2}\,5^{2/3}$$

$$= \frac{3}{2}\sqrt[3]{25}, \text{ conv.}$$

8. Let R be the region bounded by the curves $x = 2y^2$ and $x = y^2 + 1$. The **volume** of the solid generated by rotating R about the line $y = -2$ is given by

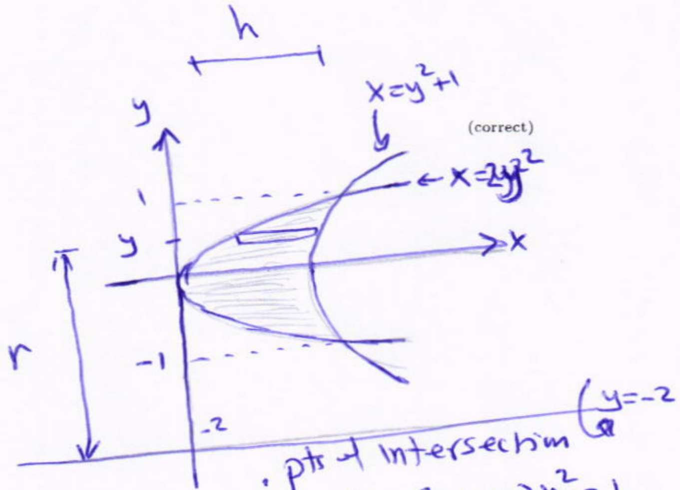
(a) $\int_{-1}^{1} 2\pi(y+2)(1-y^2)\,dy$ (correct)

(b) $\int_{-1}^{1} 2\pi(y-2)(y^2-1)\,dy$

(c) $\int_{-1}^{1} 2\pi(y-2)(1-y^2)\,dy$

(d) $\int_{-1}^{1} 2\pi y(1-y^2)\,dy$

(e) $\int_{-1}^{1} 2\pi(y+2)(y^2-1)\,dy$



. pts of intersection
$2y^2 = y^2 + 1 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1.$

. $r = y - (-2) = y + 2$

$h = (y^2+1) - 2y^2 = 1 - y^2$

$$V = \int_{-1}^{1} 2\pi \cdot (y+2)(1-y^2)\,dy$$

9. The improper integral $\int_e^{\infty} \frac{1}{x(\ln x)^2}\, dx$ is

(a) convergent and its value is 1 (correct)

(b) convergent and its value is 0

(c) convergent and its value is $\frac{1}{2}$

(d) convergent and its value is 3

(e) divergent

$$= \lim_{t\to\infty} \int_e^{t} \frac{1}{x(\ln x)^2}\, dx$$

$u = \ln x \Rightarrow du = \frac{1}{x}\, dx$

$x = e \Rightarrow u = 1$

$x = t \to u = \ln t$

$$= \lim_{t\to\infty} \int_1^{\ln t} \frac{1}{u^2}\, du$$

$$= \lim_{t\to\infty} -\frac{1}{u}\Big]_1^{\ln t}$$

$$= \lim_{t\to\infty} -\frac{1}{\ln t} - (-1) = 0 + 1 = 1$$

conv.

10. $\int 18 \sin(4x)\, \cos(5x)\, dx =$

(a) $9\cos x - \cos(9x) + C$ (correct)

(b) $18\cos x - 2\cos(9x) + C$

(c) $9\sin x - \sin(9x) + C$

(d) $9\sin x + \sin(9x) + C$

(e) $18\cos x + 2\sin(9x) + C$

$$18 \int \frac{1}{2} [\sin(4x-5x) + \sin(4x+5x)]\, dx$$

$$= 9 \int [\sin(-x) + \sin(9x)]\, dx$$

$$= 9 \int [-\sin x + \sin(9x)]\, dx$$

$$= 9 \left(\cos x - \frac{1}{9}\cos(9x)\right) + C$$

$$= 9\cos x - \cos(9x) + C$$