1. The volume of the solid obtained by rotating about the $x$-axis, the region bounded by $x+y=3$, $x=4-(y-1)^2$ is

(a) $\frac{27}{2}\pi$ (correct)

(b) $\frac{81}{4}\pi$

(c) $\frac{16}{3}\pi$

(d) $\frac{13}{4}\pi$

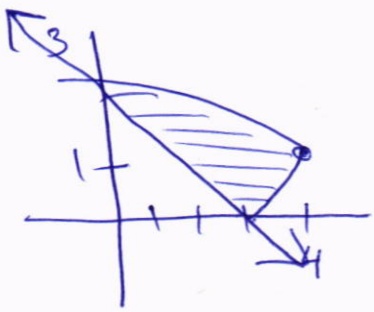
(e) $\frac{31}{3}\pi$

$4-(y-1)^2=0 \Rightarrow y-1=\pm 2$

$y=3,-1$, Vertex $(4,1)$

$x+y=3$ : $(0,3)$, $(3,0)$

#14/6.3




$$V=\int_0^3 2\pi y\left(4-(y-1)^2\right)-3+y\Big)\,dy$$

$$2\pi\int_0^3 y\left(4-(y^2-2y+1)-3+y\right)dy$$

$$2\pi\int_0^3 y(-y^2+3y)\,dy=\ldots=\frac{27}{2}\pi$$

2. The average value of $f(x)=\sec^2\left(\frac{\theta}{2}\right)$ over $\left[0,\frac{\pi}{2}\right]$ is equal to:

(a) $\frac{4}{\pi}$ (correct)

(b) $\frac{2}{\pi}$

(c) 2

(d) 4

(e) $\frac{\pi}{2}$

$$f_{avg}=\frac{1}{\frac{\pi}{2}-0}\int_0^{\pi/2}\sec^2\left(\frac{\theta}{2}\right)d\theta$$

$$=\frac{2}{\pi}\int_0^{\pi/2}\sec^2\left(\frac{\theta}{2}\right)d\theta=\frac{2}{\pi}\left[2\tan\left(\frac{\theta}{2}\right)\right]_0^{\pi/2}$$

$$=\frac{4}{\pi}\left[1-0\right]$$

#6/6.5

3. $\int_0^1 \frac{x^3}{\sqrt{4+x^2}}\,dx =$ $\int \frac{x\,x^2}{\sqrt{4-x^2}}dx$, let $u=x^2 \Rightarrow du = 2x\,dx$

(a) $\frac{16}{3} - \frac{7}{3}\sqrt{5}$ (correct)

(b) $\frac{8-\sqrt[3]{7}}{3}$

(c) $10\sqrt{5} - \frac{1}{3}$

(d) $5\sqrt{5} - \frac{8}{3}\sqrt{7}$

(e) 0

$dv = \frac{x\,dx}{\sqrt{x^2+4}} \Rightarrow v = \sqrt{4+x^2}$

$\int \frac{x^3dx}{\sqrt{4+x^2}} = x^2\sqrt{4+x^2} - 2\int x\sqrt{4+x^2}\,dx$

$= \left[x^2\sqrt{4+x^2} - \frac{2}{3}(4+x^2)^{3/2}\right]_0^1$ #34/7.1

$\left(\sqrt{5} - \frac{2}{3}(4+1)^{3/2}\right) - \left(-\frac{2}{3}\sqrt{64}\right) = \cdots = \frac{16}{3} - \frac{7\sqrt{5}}{3}$

4. $\int_0^{\frac{1}{2}} x\cos(\pi x)\,dx =$

(a) $\frac{1}{2\pi} - \frac{1}{\pi^2}$ (correct)

(b) $\frac{1-\pi}{2}$

(c) $\frac{\pi-1}{3}$

(d) $\frac{2\pi-1}{3}$

(e) $\frac{1+\pi}{\pi}$

let $u = x \Rightarrow du = dx$

$dv = \cos(\pi x)dx \Rightarrow v = \frac{1}{\pi}\sin(\pi x)$

$\int x\cos(\pi x)dx = \frac{1}{\pi}x\sin(\pi x) - \int \frac{1}{\pi}\sin(\pi x)\,dx$

$= \frac{1}{\pi}x\sin(\pi x) - \frac{1}{\pi}\left[\frac{-\cos(\pi x)}{\pi}\right]$ #23/7.1

$= \left[\frac{x}{\pi}\sin(\pi x)\right]_0^{1/2} + \left[\frac{1}{\pi^2}\cos(\pi x)\right]_0^{1/2}$

$\frac{1}{\pi}\left(\frac{1}{2} - 0\right) + \frac{1}{\pi^2}(0-1)$

5. $\int x \ln(1+x)\, dx =$

$\boxed{4.1/7.1}$

$u = \ln(1+x) \Rightarrow du = \frac{dx}{1+x}$

$dv = x\,dx \Rightarrow v = \frac{1}{2}x^2$

(a) $\frac{1}{2}(x^2-1)\ln(1+x) - \frac{1}{4}x^2 + \frac{1}{2}x + c$ ____________(correct)

(b) $x\ln(1+x) + x^2 + \ln(1+x) + c$

(c) $x^2\ln(x+1) - x\ln(1+x) + c$

(d) $x\ln(1+x) - x^2 - x + c$

(e) $\frac{1}{2}x^2\ln(x+1) - \frac{1}{2}x^2 + x + c$

$\int x\ln(1+x)\,dx = \frac{1}{2}x^2\ln(1+x) - \frac{1}{2}\int \frac{x^2}{1+x}\,dx$

Now $1+x \overline{\smash{)}x^2}$ with quotient $x-1$ $\Rightarrow \frac{x^2}{x+1} = x - 1 + \frac{1}{x-1}$

so $\int (x - 1 + \frac{1}{x-1})\,dx = \frac{1}{2}x^2 - x + \ln|x-1|$

$\int x\ln(1+x)\,dx = (\frac{1}{2}x^2 - \frac{1}{2})\ln|1+x| - \frac{1}{4}x^2 + \frac{1}{2}x + C$

6. $\int \tan^3 x\, dx =$

(a) $\frac{1}{2}\tan^2 x - \ln|\sec x| + c$ ____________(correct)

(b) $\frac{1}{4}(\tan x)^4 + c$

(c) $\tan x \sec^2 x + c$

(d) $\ln|\sec x + \tan x| + c$

(e) $\tan x - \ln|\tan x| + c$

$\boxed{\text{ex\#7/7.2}}$

$\int \tan^3 x\,dx = \int \tan^2 x \tan x\,dx$

$= \int \tan x(\sec^2 x - 1)\,dx = \int \tan x \sec^2 x - \int \tan x$

$= \frac{1}{2}(\tan x)^2 + \ln|\sec x| + C$