King Fahd University of Petroleum and Minerals
Department of Mathematics
Math 106
Final Exam
213
August 11, 2022
Net Time Allowed: 180 Minutes

MASTER VERSION

1. $\lim_{x\to 2^-} \dfrac{x+2}{x(x^2-4)} =$

   (a) $-\infty$ _____ (correct)
   (b) ∞
   (c) $\dfrac{1}{2}$
   (d) $-\dfrac{1}{2}$
   (e) 0

2. $\lim_{x\to -3} \dfrac{x^4-81}{x^2+8x+15} =$

   (a) -54 _____ (correct)
   (b) 54
   (c) ∞
   (d) $-\infty$
   (e) 0

3. Given that $f(x) = x^3 - 4x^2$. If

$$\frac{f(x+h) - f(x)}{h} = Ax^2 + Bxh + Ch^2 + Dx + Eh$$

Then $A + B + C + D + E =$

(a) -5 (correct)
(b) 5
(c) 6
(d) -6
(e) 0

4. For what values of a, $\lim_{x \to -1} \dfrac{x^2 + 2x + a}{x^2 - 2x - 3}$ does exist.

(a) 1 (correct)
(b) -1
(c) 2
(d) -2
(e) 0

5. For what value of the constant c is the function

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$$

continuous on $(-\infty, \infty)$?

(a) $\frac{2}{3}$ (correct)

(b) $-\frac{2}{3}$

(c) $\frac{3}{2}$

(d) $-\frac{3}{2}$

(e) 2

6. Find the equation of tangent line to the curve $y = \sqrt{1 + xe^{-2x}}$ at $(0, 1)$

(a) $2y = x + 2$ (correct)

(b) $2y = x + 1$

(c) $y = x + 1$

(d) $y = x + 2$

(e) $2y = 2x + 1$

7. Find the equation of the tangent line to the curve $y = x^2\, e^{-1/x}$ at $\left(1, \frac{1}{e}\right)$

(a) $y = \dfrac{3x-2}{e}$ _____ (correct)

(b) $y = \dfrac{3}{e}x - 2$

(c) $y = \dfrac{e}{3}x - 2$

(d) $y = \dfrac{x-2}{3e}$

(e) $y = \dfrac{e(x-2)}{3}$

8. If $F(x) = f(g(x))$, where $f(-2) = 8$, $f'(-2) = 4$, $f'(5) = 3$, $g(5) = -2$, $g'(5) = 6$ then $F'(5) =$

(a) 24 _____ (correct)

(b) -24

(c) -12

(d) 12

(e) 25

9. The average cost $\bar{c}$ for producing q units of a product is given by $\bar{c} = 0.00002q^2 - 0.01q + 6 + \dfrac{20,000}{q}$. Find the marginal cost for $q = 200$.

(a) 4.4 _____ (correct)
(b) 5.4
(c) 6.4
(d) 4.6
(e) 4.0

10. If $y = 2^x\, x^2$ then $\dfrac{dy}{dx} =$

(a) $x(2^x)(2 + x \ln 2)$ _____ (correct)
(b) $(2^x)(2 + x \ln 2)$
(c) $x(2 + x \ln 2)$
(d) $2x(2 + \ln 2)$
(e) $x(2^x + \ln 2)$

11. If $y = e^{-x} \ln x$, then find $\left.\dfrac{dy}{dx}\right|_{x=1} =$

(a) $\dfrac{1}{e}$ (correct)

(b) $-\dfrac{1}{e}$

(c) $-e$

(d) e

(e) 1

12. If $x\sqrt{y+1} = y\sqrt{x+1}$, find $\left.\dfrac{dy}{dx}\right|_{(3,3)} =$

(a) 1 (correct)

(b) -1

(c) 2

(d) -2

(e) 0

13. If $y = (\ln x)^{e^x}$. find $\left.\dfrac{dy}{dx}\right|_{x=e}$

(a) e^{e-1} _____(correct)
(b) e^{1-e}
(c) e^{e+1}
(d) e^{2e-1}
(e) e^{e-2}

14. If $y = \left(\dfrac{3}{x^2}\right)^x$ find $\left.\dfrac{dy}{dx}\right|_{x=1}$

(a) $3(-2+\ln(3))$ _____(correct)
(b) $3(2+\ln(3))$
(c) $2(3+\ln(3))$
(d) $2(-3+\ln(3))$
(e) $3(-2+\ln(2))$

15. The function $f(x) = 2x^3 + x^2 + 2x$ has how many critical points?

(a) 0 _____(correct)
(b) 1
(c) 2
(d) 3
(e) 4

16. On the interval $[-2, 3]$ the function

$$f(x) = 2x^3 - 3x^2 - 12x + 1$$

has an

(a) absolute maximum at $x = -1$ and absolute minimum at $x = 2$ _____(correct)
(b) absolute maximum at $x = 2$ and absolute minimum at $x = -2$
(c) absolute maximum at $x = 3$ and absolute minimum at $x = -2$
(d) absolute maximum at $x = 2$ and absolute minimum at $x = -1$
(e) absolute maximum at $x = 2$ and absolute minimum at $x = 3$

17. The graph of $f(x) = 4x^3 + 3x^2 - 6x + 1$ is

(a) increasing on $(-\infty, -1)$ and $\left(\frac{1}{2}, \infty\right)$ _____(correct)

(b) decreasing on $(-\infty, -1)$ and $\left(\frac{1}{2}, \infty\right)$

(c) increasing on $(-\infty, -1)$ and $\left(-1, \frac{1}{2}\right)$

(d) decreasing on $\left(-1, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, \infty\right)$

(e) only increasing on $\left(-1, \frac{1}{2}\right)$

18. If $f(x) = x^4 - 2x^2 + 3$ then f is concave up on the interval

(a) $\left(-\infty, -\frac{\sqrt{3}}{3}\right)$ and $\left(\frac{\sqrt{3}}{3}, \infty\right)$ _____(correct)

(b) $(-\infty, \infty)$

(c) $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

(d) $(-\infty, -\sqrt{3})$ and $(\sqrt{3}, \infty)$

(e) $(-\infty, 1/3)$ and $(1/3, \infty)$

19. The graph of

$$f(x) = \frac{x^4 + 1}{1 - x^4}$$

has

(a) two vertical asymptotes and one horizontal asysmptote _____(correct)
(b) only one vertical asymptote
(c) only two horizontal asymptote
(d) one vertical and one horizontal asymptote
(e) one vertical and two horizontal asymptotes

20. The demand equation for a product q is $p = \frac{10}{\sqrt{q}}$ using differentials, approximate the price when 24 units are demanded

(a) $\frac{51}{25}$ _____(correct)
(b) $\frac{50}{25}$
(c) $\frac{25}{50}$
(d) $\frac{25}{51}$
(e) $\frac{51}{20}$

21. The marginal revenue function is given by

$$\frac{dr}{dq} = 275 - q - 0.3q^2.$$

Find the price when 10 units of product are demanded

(a) $p = 260$ (correct)
(b) $p = 275$
(c) $p = 250$
(d) $p = 290$
(e) $p = 265$

22. $\displaystyle\int \frac{6x^2 - 11x + 5}{3x - 1}\, dx =$

(a) $x^2 - 3x + \frac{2}{3}\ln|3x - 1| + c$ (correct)
(b) $x^2 - 3 - \frac{2}{3}\ln|3x + 1| + c$
(c) $x^2 + 3x - \frac{2}{3}\ln|3x + 1| + c$
(d) $x^2 + 3x + \frac{2}{3}\ln|3x - 1| + c$
(e) $x^2 - 3x - \frac{2}{3}\ln|3x - 1| + c$

23. $\int_0^1 x^2 \sqrt[3]{7x^3+1}\,dx =$

(a) $\frac{15}{28}$ (correct)

(b) $-\frac{15}{28}$

(c) $\frac{28}{15}$

(d) $-\frac{28}{15}$

(e) $\frac{16}{28}$

24. Find the area of region bounded by $y = x^2 - 4x$, $x = 2$, $x = 6$ and x-axis

(a) 16 (correct)

(b) -16

(c) $\frac{32}{3}$

(d) $-\frac{32}{3}$

(e) $\frac{16}{3}$

25. Evaluate $\int_1^2 x^4(\ln x)^2\,dx =$

(a) $\frac{62}{125} + \frac{32}{5}(\ln 2)^2 - \frac{64}{25}\ln(2)$ _____(correct)

(b) $\frac{62}{125} - \frac{32}{5}(\ln 2)^2 + \frac{64}{25}\ln(2)$

(c) $\frac{62}{125} + \frac{5}{32}(\ln 2)^2 + \frac{25}{64}\ln(2)$

(d) $\frac{62}{125} - \frac{5}{32}(\ln 2)^2 - \frac{25}{64}\ln(2)$

(e) $\frac{62}{125} - \frac{5}{32}(\ln 2) - \frac{25}{64}\ln(2)^2$

26. Use $\int u^2\sqrt{u^2 \pm a^2}\,du = \frac{u}{8}(2u^2 \pm a^2)\sqrt{u^2 \pm a^2} - \frac{a^4}{8}\ln|u + \sqrt{u^2 \pm a^2}| + c$
to find $\int 7x^2\sqrt{3x^2 - 6}\,dx =$

(a) $\frac{7}{3\sqrt{3}}\left[\frac{3\sqrt{3}x}{4}(x^2-1)\sqrt{3x^2-6} - \frac{9}{2}\ln|\sqrt{3}x + \sqrt{3x^2-6}|\right] + c$ _____(correct)

(b) $\frac{7}{3\sqrt{3}}\left[\frac{\sqrt{3}}{4}(x^2-1)\sqrt{3x^2+6} - \frac{9}{2}\ln|\sqrt{3}x + \sqrt{3x^2+6}|\right] + c$

(c) $\frac{7}{3\sqrt{3}}\left[\frac{3\sqrt{3}}{4}x\sqrt{3x^2+6} - \frac{9}{2}\ln|\sqrt{3}x - \sqrt{3x^2+6}|\right] + c$

(d) $\frac{7}{3\sqrt{3}}\left[\frac{3\sqrt{3}}{4}x(x^2+1)\sqrt{3x^2+6} - \frac{9}{2}\ln|\sqrt{3}x + \sqrt{3x^2+6}|\right] + c$

(e) $\frac{7}{3\sqrt{3}}\left[\frac{3\sqrt{3}x}{4}(x^2-1)\sqrt{3x^2+6} - \frac{9}{2}\ln|x + \sqrt{3x^2+6}|\right] + c$

27. If $Z = e^{\sqrt{x^2+y^2}}$ find $\dfrac{\partial^2 z}{\partial y^2}$

(a) $\dfrac{Z(x^2+y^2\sqrt{x^2+y^2})}{(x^2+y^2)^{3/2}}$ (correct)

(b) $\dfrac{x^2+y^2\sqrt{x^2+y^2}}{(x^2+y^2)^{3/2}}$

(c) $\dfrac{Z(x^2\sqrt{x^2+y^2}+y^2)}{(x^2+y^2)^{3/2}}$

(d) $\dfrac{x^2\sqrt{x^2+y^2}+y^2}{(x^2+y^2)^{3/2}}$

(e) $\dfrac{Z(x^2\sqrt{x^2+y^2})}{(x^2+y^2)^{3/2}} + y^2$

28. A candy company produces two varieties of candy A and B for which the constant average costs of production are 60 SR and 70 SR respectively. The demand functions for A and B are given by

$$q_A = 5(P_B - P_A) \text{ and } q_3 = 500 + 5(P_A - 2P_B).$$

Find the selling prices P_A and P_B that maximize the company's profit

(a) $P_A = 80$ and $P_B = 85$ (correct)

(b) $P_A = 85$ and $P_B = 80$

(c) $P_A = 90$ and $P_B = 95$

(d) $P_B = 95$ and $P_B = 90$

(e) $P_A = 70$ and $P_B = 75$