KEY

1. The statement that is **wrong** about the parametric curve

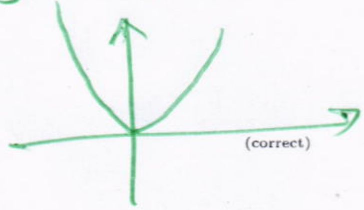
$$x = \sinh(t),\ y = -1 + \cosh(t),\ t \in (-\infty, \infty)$$

is

$\cosh^2(t) - \sinh^2(t) = 1$

$(y+1)^2 - x^2 = 1$

$y^2 + 2y - x^2 = 0.$



(a) The curve is the lower part of a hyperbola. (correct)

(b) The curve passes through the origin.

(c) The curve is symmetric about the y-axis.

(d) The cartesian equation of the curve is $x^2 - y^2 - 2y = 0$.

(e) For $t < 0$, the curve lies in the second quadrant.

$\cosh(t) \geq 1$

$-1 + \cosh(t) \geq 0$

$y \geq 0.$

Upper branch of the hyperbola $(y+1)^2 - x^2 = 1$ with vertex $(0,0)$ and openning upwards.

2. The surface area of the solid obtained by rotating the parametric curve

$$x(t) = t^3 - t,\ y = t^3 - t,\ t \in [2, 3]$$

about the x-axis is

$$S = 2\pi \int_a^b y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$= 2\pi \int_\alpha^\beta y(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

(a) $540\sqrt{2}\,\pi$ (correct)

(b) $270\sqrt{3}\,\pi$

(c) $150\sqrt{3}\,\pi$

(d) $270\sqrt{5}\,\pi$

(e) $450\sqrt{2}\,\pi$

$\frac{dx}{dt} = 3t^2 - 1$

$\frac{dy}{dt} = 3t^2 - 1$

$$= 2\pi \int_2^3 (t^3 - t)\sqrt{(3t^2-1)^2 + (3t^2-1)^2}\,dt$$

$$= 2\pi\sqrt{2} \int_2^3 \left((t^3 - t)(3t^2-1)\right)dt$$

$$= 2\sqrt{2}\pi \int_2^3 (3t^5 - 4t^3 + t)\,dt$$

$$= 2\sqrt{2}\pi \left(\frac{t^6}{2} - t^4 + \frac{t^2}{2}\right)\Big|_2^3$$

$$= 2\sqrt{2}\pi\,(270) = 540\sqrt{2}\pi.$$

3. The number of points of intersection between the polar curves

10.3

$$r = \sin(2\theta) \text{ and } r = \frac{1}{3}$$

even

is

rose with 4 leaves with tips at $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

circle centered at the origin with radius $\frac{1}{3}$.

(a) 8

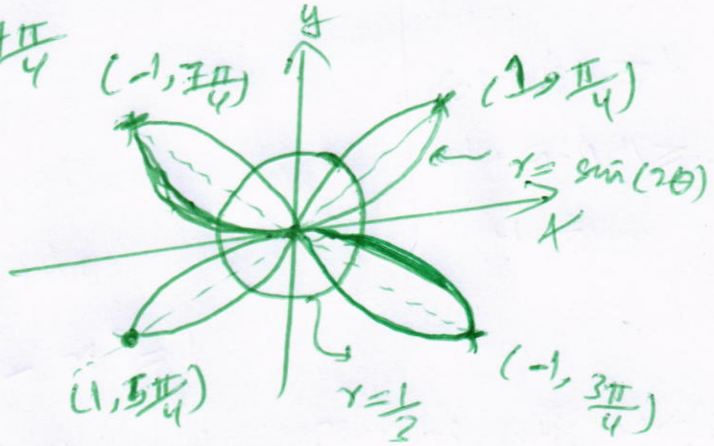
(b) 6

(c) 4

(d) 2

(e) 0

(correct)



The graphs of $r = \frac{1}{3}$ & $r = \sin(2\theta)$ is similar to the one given in Example 3 of 10.4

4. The graph of $r^2 = \sin(2\theta)$ is symmetric about

Since $r^2 \geq 0$; $0 \leq \theta \leq \frac{\pi}{2} + 2k\pi$ $+2k\pi$ $k \in \mathbb{Z}$

(a) the origin only.

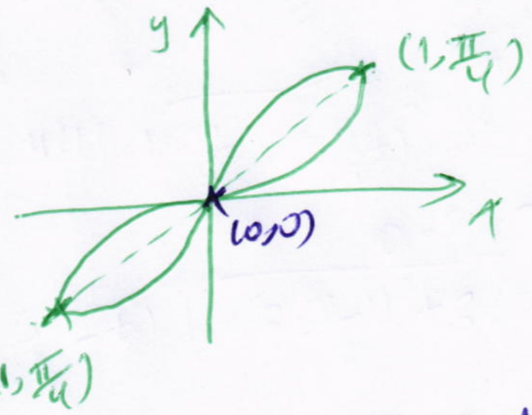
(b) the origin and the polar axis.

(c) the x-axis and the y-axis.

(d) the origin and the line $\theta = \frac{\pi}{2}$.

(e) the polar axis only.

(correct)



The graph is given in Exercise 5 of 10.4.

Symmetry about the origin $(-r, \theta)$:

$(-r)^2 = \sin(2\theta)$.

From 10.3

5. The area of the region that lies **inside** the cardioid $r = 1 - \sin(\theta)$ and **outside** the circle is $r = 1$ is

Exercise 25 of 10.4

(a) $\frac{\pi}{4} + 2$ (correct)

(b) $\frac{\pi}{2} + 1$

(c) $\frac{\pi}{3} + 3$

(d) $\frac{\pi}{6} + 2$

(e) $\frac{\pi}{2} + 3$

$A = \frac{1}{2}\int_\alpha^\beta (r_{out}^2 - r_{in}^2)\, d\theta$

$A = \frac{1}{2}\int_\pi^{2\pi} \left((1-\sin\theta)^2 - (1)^2\right) d\theta$

Symmetry

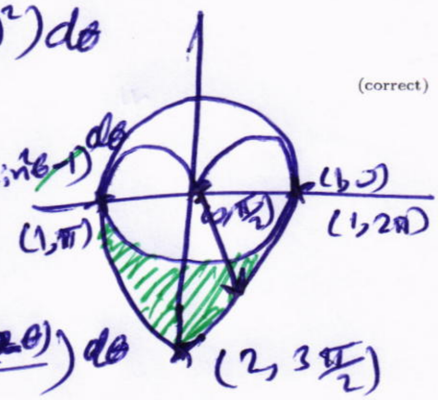
$= \frac{1}{\cancel{2}} \cdot \cancel{2}\int_\pi^{3\pi/2} (\cancel{1} - 2\sin\theta + \sin^2\theta - \cancel{1})\, d\theta$

$= \int_\pi^{3\pi/2} \left(-2\sin\theta + \frac{1-\cos(2\theta)}{2}\right) d\theta$

$= 2\cos\theta \Big|_\pi^{3\pi/2} + \left(\frac{1}{2}\theta - \frac{\sin(2\theta)}{4}\right)\Big|_\pi^{3\pi/2}$

$= 2(0-(-1)) + \frac{1}{2}\left(\frac{3\pi}{2} - \pi\right) - (0-0)$

$= 2 + \frac{\pi}{4}.$



θ	$1-\sin\theta$
0	1
$\frac{\pi}{2}$	0
π	1
$\frac{3\pi}{2}$	2
2π	0

6. The area of the region enclosed by the inner loop of $r = 1 - \sqrt{2}\sin(\theta)$ is

Similar to Exercise 35 of 10.4.

(a) $\frac{\pi - 3}{2}$ (correct)

(b) $\frac{\pi + 1}{4}$

(c) $\frac{\pi + 5}{3}$

(d) $\frac{\pi + 7}{6}$

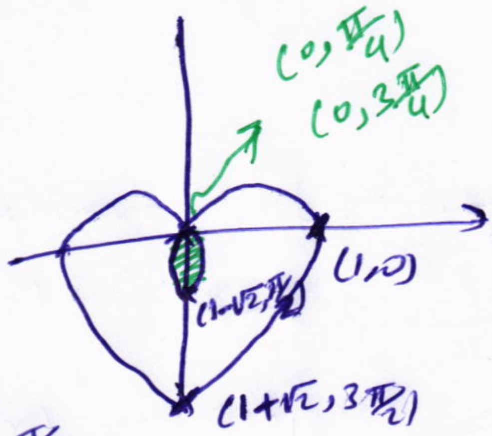
(e) $\frac{3\pi + 10}{6}$

$r = 0$, when $1 - \sqrt{2}\sin\theta = 0.$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{4} + 2k\pi$ or $\theta = \frac{3\pi}{4} + 2k\pi$ $\Big\}\ k \in \mathbb{Z}$

θ	$1-\sqrt{2}\sin\theta$
0	1
$\frac{\pi}{4}$	0
$\frac{\pi}{2}$	$1-\sqrt{2}$ (min r)
$\frac{3\pi}{4}$	0
π	1
$\frac{3\pi}{2}$	$1+\sqrt{2}$ (max r)



Using Symmetry

$A = \frac{1}{\cancel{2}} \cdot \cancel{2}\int_{\pi/4}^{\pi/2} (1 - \sqrt{2}\sin\theta)^2\, d\theta$

$= \int_{\pi/4}^{\pi/2} (1 - 2\sqrt{2}\sin\theta + 2\sin^2\theta)\, d\theta$

$= (\theta + 2\sqrt{2}\cos\theta)\Big|_{\pi/4}^{\pi/2} + \cancel{2} \cdot \int_{\pi/4}^{\pi/2} \frac{1-\cos(2\theta)}{\cancel{2}}\, d\theta$

$= \left(\frac{\pi}{2} - \frac{\pi}{4}\right) + 2\sqrt{2}\left(0 - \frac{1}{\sqrt{2}}\right) + \left(\theta - \frac{\sin(2\theta)}{2}\right)\Big|_{\pi/4}^{\pi/2}$

$= \cancel{\frac{\pi}{4}} - 2 + \left(\frac{\pi}{2} - \cancel{\frac{\pi}{4}}\right) - (0-1)/2 = \frac{\pi}{2} - \frac{3}{2}.$