1. The slope of the tangent line to the polar curve $r = 2\cos(\theta)$ at $\theta = \frac{\pi}{3}$ is

(a) $\frac{1}{\sqrt{3}}$ (correct)

(b) $\sqrt{3}$

(c) $\frac{1}{\sqrt{2}}$

(d) $\sqrt{2}$

(e) 1

$m_{\text{tangent}} = \frac{dy}{dx} = \dfrac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$

$\frac{dr}{d\theta} = -2\sin\theta$

$\left.\frac{dr}{d\theta}\right|_{\theta=\frac{\pi}{2}} = -2\cdot\frac{\sqrt{3}}{2} = -\sqrt{3}.$

$r(\frac{\pi}{3}) = 2\cdot\frac{1}{2} = 1$

$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \dfrac{-\sqrt{3}\cdot\frac{\sqrt{3}}{2} + (1)\frac{1}{2}}{-\sqrt{3}\cdot\frac{1}{2} - (1)\frac{\sqrt{3}}{2}}$

$= \dfrac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$

2. If (a, b, c) is the point of intersection between the plane $2x + y + z = 9$ and the line through $(1, 0, 1)$ and $(2, -1, 3)$, then $a + b + c =$

(a) 6 (correct)

(b) -4

(c) 0

(d) -2

(e) 10

$L \parallel \vec{v} = \langle 1, -1, 2\rangle$

$L: x = 1 + t$, $y = 0 - t$, $z = 1 + 2t$, $t \in \mathbb{R}$

setting the values of x, y & z in the equation of the plane yields:

$2x + y + z = 9$

$2(1+t) + (-t) + (1+2t) = 9$

$3t = 6$; $t = 2.$

$\therefore (a, b, c) = (1+2, -2, 1+2(2))$

$= (3, -2, 5)$. So, $a+b+c = 6.$

3. If $T = \dfrac{v}{2u+v}$, $u = pq\sqrt{r}$ and $v = p\sqrt{q}r$, then the value of $\dfrac{\partial T}{\partial q}$ when $(p,q,r) = (2,1,4)$ is

A $(u,v) = (4,8)$.

(a) $-\frac{1}{8}$

(b) $\frac{1}{8}$

(c) $\frac{1}{4}$

(d) $-\frac{1}{4}$

(e) 0

(correct)

$$\frac{\partial T}{\partial q} = \frac{\partial T}{\partial u}\cdot\frac{\partial u}{\partial q} + \frac{\partial T}{\partial v}\cdot\frac{\partial v}{\partial q}$$

$$= \frac{0-2(v)}{(2u+v)^2}\cdot p\sqrt{r} + \frac{1(2u+\cancel{v})-1(\cancel{v})}{(2u+v)^2}\cdot\frac{pr}{2\sqrt{q}}$$

$$\left.\frac{\partial T}{\partial q}\right|_A = \frac{-2(8)}{(16)(16)}\cdot(2)(2) + \frac{(2)(4)}{(16)(16)}\cdot\frac{(2)(4)}{(2)(1)}$$

$$= \frac{-1}{4} + \frac{1}{8} = \frac{-1}{8}$$

4. The number of points at which the **normal line** through the point $(1,-1,1)$ on the ellipsoid $x^2+2y^2+2z^2=5$ intersects the sphere $x^2+y^2+(z-1)^2=5$ is

(a) 2

(b) 1

(c) 0

(d) 3

(e) 4

(correct)

Consider $f(x,y,z) = x^2+2y^2+2z^2$.

normal line $\parallel \nabla f|_{(1,-1,1)} = \langle 2x, 4y, 4z\rangle|_{(1,-1,1)}$

$= \langle 2,-4,4\rangle$.

Equations of the normal line:

$P_1\ (0,1,3),\ t=-\frac{1}{2}$

$P_2\ \left(\frac{4}{3},-\frac{5}{3},\frac{5}{3}\right),\ t=\frac{1}{6}$

$$\left.\begin{aligned} x &= 1+2t \\ y &= -1-4t = -(1+4t) \\ z &= 1+4t. \end{aligned}\right\}\ t\in\mathbb{R}$$

Substituting in the equation of the sphere:

$x^2+y^2+(z-1)^2=5$; $(1+2t)^2+(1+4t)^2+(4t)^2=5$

$36t^2+12t-3=0$; $12t^2+4t-1=0$.

$t = \dfrac{-4\mp\sqrt{16-(4)(12)(-1)}}{24} = \dfrac{-4\mp\sqrt{64}}{24}$

$t=-\frac{1}{2}$ or $t=\frac{1}{6}$

5. If $(1,1)$ is a critical point of $f(x,y)=x^4+y^4-4xy+1$, then

(a) f has a local minimum at $(1,1)$ (correct)

(b) f has a local maximum at $(1,1)$

(c) f has a saddle point at $(1,1)$

(d) f has an absolute maximum at $(1,1)$

(e) none of the above

$f_x = 4x^3 - 4y$

$f_y = 4y^3 - 4x$

$f_x(1,1) = 0 = f_y(1,1)$

$f_{xx} = 12x^2;\ f_{yy} = 12y^2;\ f_{xy} = -4 = f_{yx}$

$D(x,y) = f_{xx}\cdot f_{yy} - (f_{xy})^2 = 144x^2y^2 - 16.$

$D(1,1) = 144 - 16 > 0.$ Since $f_{xx}(1,1) = 12 > 0$, $f(1,1)$ is local minimum.

6. The absolute minimum m and the absolute maximum M of $f(x,y)=x^2+y^2+x^2y+4$ on $D=\{(x,y)\,|\,|x|\le 1,\ |y|\le 1\}$ are

(x,y)	$f(x,y)$
$(0,0)$	4
$(0,1)$	5
$(1,-\frac{1}{2})$	19/4
$(-1,-\frac{1}{2})$	19/4
$(-1,-1)$	5
$(1,1)$	7
$(-1,1)$	7
$(1,-1)$	5

absolute min.

absolute max

(a) $m=4,\ M=7$ (correct)

(b) $m=5,\ M=7$

(c) $m=4,\ M=9$

(d) $m=4,\ M=6$

(e) $m=5,\ M=9$

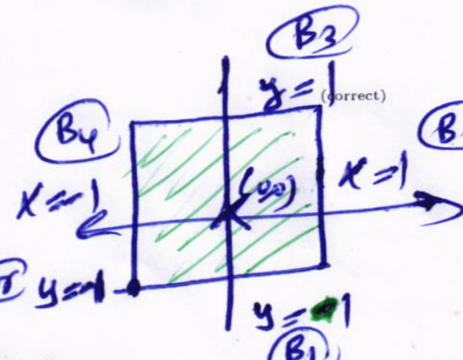
$f_x = 2x(1+y)$

$f_y = 2y + x^2.$

$f_x = 0 \Leftrightarrow x=0$ or $y=-1$

$x=0:\ f_y=0 \Leftrightarrow y=0$

$y=-1:\ f_y=0 \Leftrightarrow x=\mp\sqrt{2}$ (rejected)



$\therefore f$ has only one critical point $(0,0)$.

$B_1: \boxed{y=-1}$ $f(x,-1) = f_1(x) = 5$ constant function.

$B_2: \boxed{y=1}$ $f(x,1) = f_2(x) = 2x^2+5;\ x\in[-1,1]$

$f_2'(x) = 2x;\ f_2'(x)=0 \Leftrightarrow x=0.$ $(0,1)$

$B_3: x=\mp 1$

B_4

$f_3(y) = f(\mp 1, y) = y^2+y+5;\ y\in[-1,1]$

$f_3'(y) = 2y+1;\ f_3'(y)=0 \Leftrightarrow y=-\frac{1}{2}.$

$(1,-\frac{1}{2})$ $(-1,-\frac{1}{2})$