King Fahd University of Petroleum and Minerals
Department of Mathematics
Math 201
Major Exam II
212
March 23, 2022
Net Time Allowed: 90 Minutes

MASTER VERSION

Key

1. The distance from the point $A(1,1,1)$ to the plane that contains the point $B(1,-2,1)$ and is perpendicular to the vector from the origin to B, is

(a) $\sqrt{6}$
(b) $\sqrt{7}$
(c) $\sqrt{5}$
(d) 2
(e) 1

$\overrightarrow{OB} = \langle 1, -2, 1 \rangle$

$\Rightarrow$ eq^n of the plane:

$(x-1) - 2(y+2) + 1(z-1) = 0$

$\Rightarrow \boxed{x - 2y + z - 6 = 0}$

$$\text{distance} = \frac{|1 - 2 + 1 - 6|}{\sqrt{1+4+1}} = \frac{6}{\sqrt{6}} = \sqrt{6}$$

2. The equation of the plane that contains the two lines

$L_1: \; x = 1 + t, \quad y = 1 - t, \quad z = 2t$ $\qquad \vec{v}_1 = \langle 1, -1, 2 \rangle$

$L_2: \; x = 2 - s, \quad y = s, \quad z = 2$ $\qquad \vec{v}_2 = \langle -1, 1, 0 \rangle$

is

(a) $x + y - 2 = 0$
(b) $3x - y + z = 1$
(c) $x - y + 3 = 0$
(d) $x - 5y - z = 4$
(e) $2x - 2y + 7z = 3$

$$\vec{n} = \vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 2 \\ -1 & 1 & 0 \end{vmatrix} = \langle -2, -2, 0 \rangle$$

$P = (1,1,0)$

$\Rightarrow$ eq^n of the plane is:

$-2(x-1) - 2(y-1) = 0$

$\Rightarrow -2x + 2 - 2y + 2 = 0$

$\Rightarrow \boxed{x + y - 2 = 0}$

3. The surface S defined by

$$x^2 + 2z^2 - 6x - y = -10$$

represents

$(x^2-6x+9)+2z^2=y-10+9$

$\Rightarrow \boxed{(x-3)^2+2z^2=(y-1)}$

elliptic paraboloid

(a) an elliptic paraboloid with axis parallel to the y-axis
(b) a cone with axis parallel to the z-axis
(c) a hyperboloid of one-sheet with axis parallel to the y-axis
(d) an ellipsoid centered at $(-1, 2, 1)$
(e) a hyperbolic paraboloid with axis parallel to the x-axis

4. Consider the following statements about the surface

$$S: \; z = \sqrt{x^2 + 2y^2 - 4y + 2x + 3}$$

(I) It represents a hyperboloid of one-sheet
(II) It has a vertex at $(-1, 1, 0)$
(III) Its axis is parallel to the z-axis

Which of the above statements are true about S?

$z^2=(x^2+2x+1)+2(y^2-2y+1)+3-3$

$\boxed{z^2=(x+1)^2+2(y-1)^2}$

(a) II and III
(b) III only
(c) I, II and III
(d) II only
(e) I and II

. upper half of a cone with z-axis as axis of symmetry and vertex $(-1,1,0)$

5. The domain of the function

$$f(x,y) = \frac{1}{\sqrt{9-x^2-y^2}}$$

is

$9 - x^2 - y^2 > 0$

$\Rightarrow x^2 + y^2 < 9$

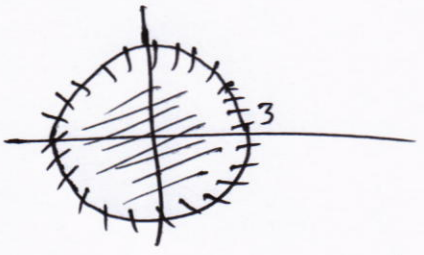
(a) the region inside the circle $x^2 + y^2 = 9$

(b) the circle $x^2 + y^2 = 9$

(c) the region outside the circle $x^2 + y^2 = 9$

(d) the region inside and on the circle $x^2 + y^2 = 9$

(e) the region outside and on the circle $x^2 + y^2 = 9$



boundary not included

6. $\lim_{(x,y)\to(1,1)} \frac{y-x}{(1-y)+3\ln x} =$

(a) does not exist

(b) 0

(c) -1

(d) 1

(e) ∞

Along the line $x = 1$:

$\lim_{(1,y)\to(1,1)} \frac{y-1}{1-y} = -1$ — (1)

Along the line $y = 1$:

$\lim_{(x,1)\to(1,1)} \frac{1-x}{3\ln x} = \lim_{x\to 1} \frac{-1}{\frac{3}{x}}$

$= \frac{-1}{3}$ — (2)

$(1) \neq (2)$