King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 201

Final Exam

212

May 16, 2022

Net Time Allowed: 135 Minutes

MASTER VERSION

1. The absolute minimum of

$$f(x,y) = x^2 - y^2 + 4y$$

on the disc $x^2 + y^2 \leq 9$ is equal to

(a) -21 (correct)
(b) 4
(c) 11
(d) 0
(e) 3

2. Let C be a curve given by the polar equation $r = \sec\theta(2 + \tan\theta)$. The slope of the tangent line to the curve C at $\theta = \dfrac{\pi}{4}$ is

(a) 4 (correct)
(b) 2π
(c) π
(d) 2
(e) -2

3. If M and m are the maximum and minimum values of

$$f(x, y) = xy$$

subject to $4x^2 + y^2 = 8$, then $M - m =$

(a) 4 (correct)

(b) 0

(c) 1

(d) 2

(e) -3

4. The average value of

$$f(x, y) = x \sin y$$

over the region $R = \{(x, y) \,|\, 0 \le x \le 1,\ 0 \le y \le \pi\}$ is

(a) $\dfrac{1}{\pi}$ (correct)

(b) 0

(c) $\dfrac{-2}{\pi}$

(d) $\dfrac{\pi}{2}$

(e) $\dfrac{2}{\pi}$

5. $\displaystyle\int_0^{\ln 2}\int_{e^y}^{2} \frac{y^4+3}{x^2+1}\ dxdy =$

(a) $\displaystyle\int_1^{2}\int_0^{\ln x} \frac{y^4+3}{x^2+1}\ dydx$ (correct)

(b) $\displaystyle\int_0^{2}\int_1^{\ln x} \frac{y^4+3}{x^2+1}\ dydx$

(c) $\displaystyle\int_1^{\ln x}\int_1^{2} \frac{y^4+3}{x^2+1}\ dydx$

(d) $\displaystyle\int_1^{2}\int_1^{\ln x} \frac{x^4+3}{y^2+1}\ dxdy$

(e) $\displaystyle\int_0^{\ln 2}\int_{e^x}^{2} \frac{y^4+3}{x^2+1}\ dxdy$

6. The volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the xy-plane, and inside the cylinder $x^2 + y^2 = 2y$ is given by

(a) $\displaystyle\int_0^{\pi}\int_0^{2\sin\theta} r^3\ drd\theta$ (correct)

(b) $\displaystyle\int_0^{2\pi}\int_0^{2\sin\theta} r^3\ drd\theta$

(c) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{2\cos\theta} r^3\ drd\theta$

(d) $\displaystyle\int_0^{\pi}\int_0^{\sin\theta} r^2\ drd\theta$

(e) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{2\cos\theta} r^2\ drd\theta$

7. The volume of the solid bounded by $z = x^2 + y^2$, and $z = 2 - x^2 - y^2$ is equal to

(a) π (correct)

(b) $\dfrac{\pi}{2}$

(c) 3π

(d) 4π

(e) $\dfrac{\pi}{4}$

8. If $E = \{(x, y, z) \,|\, 0 \le x \le 1,\ 0 \le y \le 1, 0 \le z \le 1\}$, then $\displaystyle\iiint_E xyz^2 dV =$

(a) $\dfrac{1}{12}$ (correct)

(b) $\dfrac{1}{26}$

(c) $\dfrac{1}{3}$

(d) $\dfrac{1}{2}$

(e) $-\dfrac{1}{6}$

9. The volume of the tetrahedron T bounded by the planes $x + 2y + z = 2$, $x = 2y$, $x = 0$ and the xy-plane is equal to

(a) $\frac{1}{3}$ (correct)

(b) $\frac{2}{3}$

(c) $\frac{4}{3}$

(d) $\frac{1}{2}$

(e) $\frac{1}{6}$

10. The volume of the solid bounded by $x^2 + (y-1)^2 = 1$, and the planes $z = 0$ and $x + z = 2$ is given by

(a) $\int_0^{\pi} \int_0^{2\sin\theta} \int_0^{2-r\cos\theta} r \; dzdrd\theta$ (correct)

(b) $\int_0^{2\pi} \int_0^{2\cos\theta} \int_0^{1-r\cos\theta} \; dzdrd\theta$

(c) $\int_0^{\frac{\pi}{2}} \int_0^{2\sin\theta} \int_0^{1-r\cos\theta} \; dzdrd\theta$

(d) $\int_0^{\pi} \int_0^{2\sin\theta} \int_0^{1-r\cos\theta} \; dzdrd\theta$

(e) $\int_0^{2\pi} \int_0^{2\cos\theta} \int_0^{2} r \; dzdrd\theta$

11. The volume of the solid E that lies above the cone $\phi = \dfrac{\pi}{6}$ and below the sphere $\rho = 2\cos\phi$ is

(a) $\dfrac{7\pi}{12}$ (correct)

(b) $\dfrac{\pi}{3}$

(c) $\dfrac{\pi}{4}$

(d) $\dfrac{5\pi}{11}$

(e) $\dfrac{3\pi}{4}$

12. $\displaystyle\int_0^{2\pi}\int_0^{\pi}\int_0^{\frac{1-\cos\phi}{2}} \rho^2 \sin\phi \; d\rho d\phi d\theta =$

(a) $\dfrac{\pi}{3}$ (correct)

(b) $\dfrac{\pi}{2}$

(c) $\dfrac{\pi}{7}$

(d) $\dfrac{5\pi}{4}$

(e) π

13. The area of the region enclosed by one loop of the curve $r = \sin 3\theta$ is

(a) $\dfrac{\pi}{12}$ (correct)

(b) $\dfrac{\pi}{2}$

(c) $\dfrac{\pi}{7}$

(d) $\dfrac{5\pi}{3}$

(e) 2π

14. $\displaystyle\lim_{(x,y)\to(1,1)} \frac{xy^2 - 1}{y - 1} =$

(a) does not exist (correct)

(b) 0

(c) -1

(d) 1

(e) ∞

15. A point on the cone $x^2 + y^2 - z^2 = 0$, where the tangent plane is parallel to the plane $3x + 4y + 5z = 0$ is

(a) $(3, 4, -5)$ (correct)

(b) $(3, 1, 1)$

(c) $(2, 4, 5)$

(d) $(2, 3, 3)$

(e) $(1, 1, -5)$

16. The volume of the solid bounded by the cylinder $x^2 + y^2 - 2y = 0$ on the lateral sides and bounded on top and bottom by the sphere $x^2+y^2+z^2 = 4$ is given by

(a) $\int_0^{\pi} \int_0^{2\sin\theta} \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r \; dzdrd\theta$ (correct)

(b) $\int_0^{\frac{\pi}{2}} \int_0^{2\sin\theta} \int_{-\sqrt{2-r^2}}^{\sqrt{2-r^2}} r \; dzdrd\theta$

(c) $\int_0^{\frac{\pi}{2}} \int_0^{\sin\theta} \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} dzdrd\theta$

(d) $\int_0^{\pi} \int_0^{2\cos\theta} \int_0^{\sqrt{4-r^2}} r \; dzdrd\theta$

(e) $\int_0^{\pi} \int_0^{2\cos\theta} \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} dzdrd\theta$

17. The distance from the point $P(5, 6, -1)$ to the line

$$L: \quad x = 2 + 8t, \quad y = 4 + 5t, \quad z = -3 + 6t$$

is equal to

(a) $\dfrac{3}{5\sqrt{5}}$ (correct)

(b) $\dfrac{1}{\sqrt{5}}$

(c) $\dfrac{3}{\sqrt{5}}$

(d) $\dfrac{2}{\sqrt{5}}$

(e) $\dfrac{4}{3\sqrt{5}}$

18. The normal line of the surface $4x^2 + y^2 + 2z = 6$ at the point $P(-1, 2, -1)$ passes through the point

(a) $(-9, 6, 1)$ (correct)

(b) $(7, 2, 1)$

(c) $(5, -6, 1)$

(d) $(3, 3, 2)$

(e) $(9, 9, 6)$

19. Let C be a curve given by the parametric equations

$$x = \frac{1}{t}, \qquad y = t - \sin(\pi t), \qquad t > 0.$$

The slope of the tangent line to the curve C at $t = 1$ is

(a) $-1 - \pi$ (correct)
(b) $2 - \pi$
(c) $2 + \pi$
(d) π
(e) $-2 + 2\pi$

20. The function

$$f(x, y) = x^3 + y^3 - 6xy$$

has

(a) one local minimum (correct)
(b) one local maximum
(c) two saddle points
(d) two local maxima
(e) no saddle points

21. If $z = \tan^{-1}(\frac{u^2}{\sqrt{v}})$, where $u = 2y - x$ and $v = 3x - y$.

Then $\frac{\partial z}{\partial y}$ at $(x, y) = (2, 2)$ is

(a) $\frac{17}{20}$ (correct)

(b) $\frac{7}{2}$

(c) $\frac{14}{5}$

(d) 2

(e) $\frac{2}{3}$