1. The parametric curve

$$x = \sin(t),\ y = \sin^2(t),\ -\frac{\pi}{3} \leq t \leq \frac{8\pi}{3}$$

passes through the origin

$x = 0 = y.$

(a) 3 times
(b) 6 times
(c) 5 times
(d) 4 times
(e) 2 times

$\sin(t) = 0 \iff t = n\pi,\ n \in \mathbb{Z}$ (correct)

In the given interval:

$t = 0,\ t = \pi,\ t = 2\pi$

For all these values $y = \sin^2(t) = 0.$

2. A cartesian equation of the parametric curve

$$x = \sin(t),\ y = \cot^2(t),\ \frac{\pi}{2} \leq t < \pi$$

is given by

$\cot^2(t) + 1 = \csc^2(t) = \frac{1}{\sin^2(t)}$

$y + 1 = \frac{1}{x^2}$ (correct)

$x^2(y+1) = 1.$

(a) $x^2(y+1) = 1,\ 0 < x \leq 1$
(b) $y = x + \frac{1}{x},\ 0 < x \leq 1$
(c) $x^2 y = 1 - x^2,\ -1 < x \leq 0$
(d) $xy = 1 - \frac{1}{x},\ 0 < x \leq 1$
(e) $y = \sqrt{x}(1-x),\ \frac{1}{2} < x \leq 1$

$\frac{\pi}{2} \leq t < \pi$

$0 < \sin(t) \leq 1$

$0 < x \leq 1$

3. The length of the parametric curve

$$x = \cos(t) + t\sin(t),\ y = \sin(t) - t\cos(t),\ t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

is

(a) $\frac{\pi^2}{4}$

(b) $\frac{\pi}{3}$

(c) $\frac{3}{\pi^2}$

(d) π

(e) $\frac{2\pi}{9}$

$$L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$= \int_{-\pi/2}^{\pi/2} \sqrt{(-\cancel{\sin t} + \cancel{\sin t} + t\cos t)^2 + (\cancel{\cos t} - \cancel{\cos t} + t\sin t)^2}\, dt$$

(correct)

$$= \int_{-\pi/2}^{\pi/2} \sqrt{t^2(\cos^2(t) + \sin^2(t))}\, dt$$

$$= \int_{-\pi/2}^{\pi/2} \sqrt{t^2}\, dt = \int_{-\pi/2}^{\pi/2} |t|\, dt = 2\int_0^{\pi/2} t\, dt$$

$$= \cancel{2}\cdot\frac{t^2}{\cancel{2}}\Big|_0^{\pi/2}$$

$$= \frac{\pi^2}{4}$$

4. The parametric curve

$$x = t - \ln(t),\ y = t + \ln(t)$$

is concave up on

(a) $(0, 1)$

(b) $(-\infty, 0) \cup (1, \infty)$

(c) $(1, \infty)$

(d) $(-\infty, 1)$

(e) $(-1, 0)$

$$y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1 + \frac{1}{t}}{1 - \frac{1}{t}} = \frac{t+1}{t-1}$$

$$\frac{d^2y}{dx^2} = \frac{d(y')/dt}{dx/dt} = \frac{\frac{1(\cancel{t}-1) - 1(\cancel{t}+1)}{(t-1)^2}}{\frac{(t-1)}{t}}$$

(correct)

$$= \frac{-2t}{(t-1)^3}$$

y'' - - - ++++ D.E - - -

0 1

5. The graph of the polar equation

$$r = 2(\cos(\theta) - \sin(\theta))$$

is a circle with

(a) center $(1, -1)$ and radius $\sqrt{2}$
(b) center $(-1, 1)$ and radius $\sqrt{2}$
(c) center $(1, 1)$ and radius $\sqrt{2}$
(d) center $(-1, -1)$ and radius $\sqrt{2}$
(e) center $(-1, -1)$ and radius 2

$r^2 = 2(r\cos(\theta) - r\sin(\theta))$

$x^2 + y^2 = 2x - 2y.$

$(x-1)^2 + (y+1)^2 = 2.$

(correct)

center $(1,-1)$

radius $\sqrt{2}$.

6. The slope of the tangent line to the curve

$$r = 1 + 2\sin(\theta)\cos(\theta) = 1 + \sin(2\theta)$$

at $\theta = \frac{\pi}{4}$ is

(a) -1
(b) 1
(c) 0
(d) 2
(e) -2

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

(correct)

$$= \frac{\cancel{2\cos(2\theta)}^{\text{zero}} \cdot \sin\theta + (1+\sin(2\theta))\cos\theta}{\cancel{2\cos(2\theta)}_{\text{zero}}\cos\theta - (1+\sin(2\theta))\sin\theta}$$

$$= -\cot(\theta)$$

$$m_{\text{tangent}} = \left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = -\cot(\pi/4) = -1.$$

7. The length of the polar curve

$$r = \frac{e^{2\theta}}{\sqrt{5}},\ 0 \le \theta \le \pi$$

is

(a) $\dfrac{e^{2\pi}-1}{2}$

(b) $e^{2\pi}-1$

(c) $\dfrac{e^{2\pi-1}}{5}$

(d) $e^{2\pi}-5$

(e) $e^{2\pi+1}$

$$\left(\frac{dr}{d\theta}\right)^2 = \left(\frac{2e^{2\theta}}{\sqrt{5}}\right)^2 = \frac{4}{5}e^{4\theta}$$

$$r^2 = \left(\frac{e^{2\theta}}{\sqrt{5}}\right)^2 = \frac{e^{4\theta}}{5}$$

(correct)

$$L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$= \int_0^\pi \sqrt{\frac{e^{4\theta}}{5} + \frac{4e^{4\theta}}{5}}\, d\theta$$

$$= \int_0^\pi e^{2\theta}\, d\theta = \frac{e^{2\theta}}{2}\Big|_0^\pi$$

$$= \frac{e^{2\pi}-1}{2}$$

8. The area of the region bounded by the circle $r = 2\sin(\theta)$ for $\frac{\pi}{12} \le \theta \le \frac{\pi}{2}$ is

(a) $\dfrac{5\pi+3}{12}$

(b) $\dfrac{5\pi-3}{12}$

(c) $\dfrac{5\pi-3\sqrt{3}}{12}$

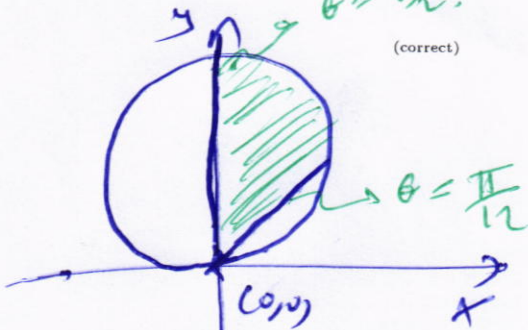
(d) $\dfrac{5\pi+3\sqrt{3}}{12}$

(e) $\dfrac{5\pi}{12}$

$r^2 = 2r\sin\theta$ $\quad x^2+y^2=2y$

$x^2+(y-1)^2=1$

(correct)



$$A = \frac{1}{2}\int_\alpha^\beta r^2\, d\theta = \frac{1}{2}\int_{\pi/12}^{\pi/2} (2\sin\theta)^2\, d\theta$$

$$= 2\int_{\pi/12}^{\pi/2} \sin^2\theta\, d\theta = \cancel{2}\int_{\pi/12}^{\pi/2} \frac{1-\cos(2\theta)}{\cancel{2}}\, d\theta$$

$$= \left(\theta - \frac{\sin(2\theta)}{2}\right)\Big|_{\pi/12}^{\pi/2} = \left(\frac{\pi}{2} - \frac{\pi}{12}\right) - \frac{\overset{\text{zero}}{\sin(\pi)} - \sin\left(\frac{\pi}{6}\right)}{2}$$

$$= \frac{5\pi}{12} + \frac{1}{4} = \boxed{\frac{5\pi+3}{12}}.$$