1. If the symmetric equations of the line through the point $(2, 3, -7)$ and orthogonal to the plane $x - y + 5z = 1$ are given by

$$\frac{x-2}{2} = \frac{y-3}{b} = \frac{z+7}{c},$$

then $b + c =$

(a) 8 (correct)
(b) 4
(c) $-8$
(d) $-4$
(e) 13

$L \parallel \langle 1, -1, 5 \rangle$

$$\frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-(-7)}{5}$$

$$\therefore \frac{x-2}{2} = \frac{y-3}{-2} = \frac{z+7}{10}$$

$b + c = -2 + 10 = 8$

2. The distance between the two planes

$$x - 3y + 4z = 5 \text{ and } \boxed{x - 3y + 4z = 3}$$

is

$Ax + By + Cz + d = 0.$
$x - 3y + 4z + (-3) = 0.$

The distance between the two parallel planes is the same as the distance between any point on the first plane and the second plane.

(a) $\frac{2}{\sqrt{26}}$ (correct)
(b) $\frac{8}{\sqrt{26}}$
(c) $\frac{3}{\sqrt{13}}$
(d) $\frac{2}{5}$
(e) $\frac{1}{3}$

Consider $P(5, 0, 0)$ on the first plane.

$$d = \frac{|Ax_0 + By_0 + Cz_0 + d|}{\sqrt{A^2 + B^2 + C^2}}$$

$$= \frac{|1(5) + 0 + 0 + (-3)|}{\sqrt{(1)^2 + (-3)^2 + (4)^2}} = \frac{2}{\sqrt{26}}$$

3. The line passing through the point $(1,2,3)$ and orthogonal to the plane $x+3y+z=21$ passes also through the point

(a) $(0,-1,2)$ (correct)

(b) $(2,5,0)$

(c) $(3,0,5)$

(d) $(-2,-7,1)$

(e) $(-1,1,2)$

$L \parallel \langle 1,3,1 \rangle$.

$x = x_0 + at = 1+t$
$y = y_0 + bt = 2+3t$ $\quad t \in \mathbb{R}$.
$z = z_0 + ct = 3+t$

The line passes through $(0,-1,2)$ at $t=-1$.

No other point of the given ones satisfies the parametric equations of $L$ for any $t \in \mathbb{R}$.

4. The intersection of the paraboloids $z=x^2+y^2$ and $z=4-x^2-y^2$ is

(a) a circle (correct)

(b) an ellipse that is not a circle

(c) a plane

(d) a straight line

(e) a single point

$x^2+y^2 = 4-(x^2+y^2)$

$2(x^2+y^2) = 4$.

$\boxed{x^2+y^2=2}$

Circle in the plane $z=2$ with center $(0,0,2)$ & radius $\sqrt{2}$.

5. The quadratic equation

$$-2x^2 + 8x + y + 5z^2 = 8$$

represents

$-2(x^2 - 4x + 4) + y + 5z^2 = 8 - 8$

(a) a hyperbolic paraboloid (correct)

(b) an elliptic paraboloid

(c) an elliptic cone

(d) a hyperboloid of one sheet

(e) a hyperboloid of two sheets

$-2(x-2)^2 + y + 5z^2 = 0.$

$y = 2(x-2)^2 - 5z^2.$

$$\boxed{y = \frac{(x-2)^2}{\left(\frac{1}{\sqrt{2}}\right)^2} - \frac{z^2}{\left(\frac{1}{\sqrt{5}}\right)^2}}$$

6. Consider the following statements about the surface

$$x = \sqrt{4y^2 + z^2 - 8y + 2z + 9} \geq 0$$

(I) Its graph consists of two sheets

(II) It has a vertex at $(0, 1, -1)$ (does not satisfy the eqn.)

(III) Its axis is parallel to the $x$-axis

Which of the statement(s) above is (are) true about this surface?

$x \geq 0$ & $x^2 = 4(y^2 - 2y + 1) + (z+1)^2 + 4$

(a) Only III (correct)

(b) II and III

(c) I, II and III

(d) I and III

(e) Only II

$x \geq 0$ & $x^2 - \frac{(y-1)^2}{\left(\frac{1}{2}\right)^2} - (z+1)^2 = 4$

$$\boxed{\boxed{x \geq 0} \ \& \ \frac{x^2}{(2)^2} - (y-1)^2 - \frac{(z+1)^2}{4} = 1}$$

One sheet of the hyperboloid of two sheets with axis parallel to the $x$-axis.

7. The range of

$$f(x,y,z) = \frac{\sqrt{1-z^2}}{4+\sqrt{1-(x^2+y^2)}} \geq 0. \quad \text{Clear}$$

the min. is equal to 0 for $z = \mp 1$

is

(a) $\left[0, \frac{1}{4}\right]$ (correct)
(b) $\left[0, \frac{1}{2}\right]$
(c) $[-1, 1]$
(d) $\left[0, \frac{1}{5}\right]$
(e) $[-1, 2]$

The max. is obtained when the numerator is max. & the denomerator is min.

N: $\sqrt{1-z^2} \leq 1$ (max. when $z=0$).

D: $4+\sqrt{1-(x^2+y^2)} \geq 4$ (min. when $x^2+y^2=1$).

$\therefore \ f(x,y,z) \leq \frac{\max(N)}{\min(D)} \leq \frac{1}{4}$.

8. $\lim\limits_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2-xy+y^2}$

$$= \lim_{(x,y)\to(0,0)} \frac{(x+y)\cancel{(x^2-xy+y^2)}}{\cancel{x^2-xy+y^2}}$$

(a) equals 0 (correct)
(b) equals 1
(c) equals $-1$
(d) equals 2
(e) does not exist

$$= \lim_{(x,y)\to(0,0)} (x+y) = 0+0 = 0.$$