1. The area of the surface obtained by rotating the curve

$$x = \cos^3(t),\ y = \sin^3(t),\ 0 \le t \le \frac{\pi}{2}$$

about the x-axis is

$\frac{dx}{dt} = 3\cos^2(t)\,(-\sin t)$

$\frac{dy}{dt} = 3\sin^2(t)\cdot(\cos t)$

(a) $\frac{6\pi}{5}$ ________________ (correct)

(b) $\frac{4\pi}{5}$

(c) $\frac{3\pi}{5}$

(d) $\frac{2\pi}{5}$

(e) $\frac{\pi}{5}$

$S = 2\pi\int_\alpha^\beta y(t)\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

$= 2\pi\int_0^{\pi/2} \sin^3(t)\sqrt{9\cos^4(t)\sin^2(t)+9\sin^4(t)\cdot\cos^2(t)}\,dt$

$= 2\pi(3)\int_0^{\pi/2} \sin^3(t)\cdot|\cos(t)||\sin(t)|\overset{1}{\cancel{\sqrt{\cos^2(t)+\sin^2(t)}}}\,dt$

$= 6\pi\int_0^{\pi/2} \overset{u}{\sin^4(t)}\cdot\overset{du}{\cos(t)\,dt}$

$= 6\pi \left.\frac{\sin^5(t)}{5}\right|_0^{\pi/2} = 6\pi\left(\frac{1}{5}-0\right) = \frac{6\pi}{5}$

2. The polar curve

$$r = 1 + 2\cos(\theta)$$

is a

$r = a + b\cos(\theta)$
$|b| > |a|$ limacon with inner loop.

(a) limacon with an inner loop ________________ (correct)

(b) circle

(c) cardioid

(d) rose with 3 leaves

(e) rose with 6 leaves

$\boxed{\text{Details}}$

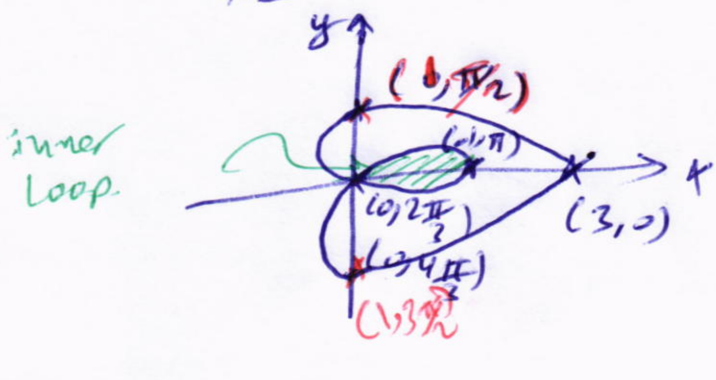
in $[0, 2\pi]$

$r = 0 \Leftrightarrow \theta = 2\pi/3$ or $\frac{4\pi}{3}$.

$2\pi/3 \le \theta \le \pi$ | $\pi \le \theta \le \frac{4\pi}{3}$

$-1 \le \cos\theta \le \frac{-1}{2}$ | $-1 \le \cos\theta \le \frac{-1}{2}$

$-1 \le r \le 0$ | $-1 \le r \le 0$



3. The number of vectors $\vec{v}$ such that

$$\vec{v} \times \underbrace{< 1, -1, 3 >}_{\vec{u}} = < 1, 1, 2 >$$

is

$\vec{u} \perp (\vec{v} \times \vec{u})$

So we should have

(a) 0 ______________________________(correct)

(b) 1

(c) 2

(d) 3

(e) ∞

$\vec{u} \cdot (\vec{v} \times \vec{u}) = 0$.

However, $\vec{u} \cdot \langle 1,1,2 \rangle$

$= \langle 1,-1,3 \rangle \cdot \langle 1,1,2 \rangle$

$= 1 - 1 + 6 = 6 \neq 0$.

$\therefore \langle 1,1,2 \rangle$ cannot be $\vec{v} \times \vec{u}$ for any vector $\vec{v}$.

4. If θ is the angle between the planes

$$x + 2y + z = 1 \text{ and } 2x - y + z = 3$$

then $\cos(\theta) =$

$\vec{n_1} = \langle 1, 2, 1 \rangle$

$\vec{n_2} = \langle 2, -1, 1 \rangle$

(a) $\frac{1}{6}$ ______________________________(correct)

(b) 0

(c) $-\frac{1}{6}$

(d) $-\frac{1}{3}$

(e) $\frac{1}{3}$

$$\cos(\theta) = \frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}||\vec{n_2}|}$$

$$= \frac{\cancel{2} \cancel{-2} + 1}{\sqrt{1+4+1}\sqrt{4+1+1}}$$

$$= \frac{1}{\sqrt{6}\sqrt{6}} = \frac{1}{6}$$

5. The intersection between the paraboloid $z = x^2 + y^2$ and the plane $2x + 2y - z = 2$ consists of

(a) one point
(b) a circle with radius 1
(c) a straight line
(d) two points
(e) a parabola

$z = x^2 + y^2 = 2x + 2y - 2$

$x^2 - 2x + y^2 - 2y = -2.$

$(x^2 - 2x + 1) + (y^2 - 2y + 1) = -2 + 2$

$(x-1)^2 + (y-1)^2 = 0.$

$(x, y) = (1, 1).$

6. The function

$$f(x, y) = x^2 + xy + y^2 + 8y$$

has at the **critical point** $P\left(\frac{8}{3}, -\frac{16}{3}\right)$

(a) a local minimum
(b) a local maximum
(c) a saddle point
(d) an absolute maximum
(e) a zero discriminant (whence the 2nd derivative test fails)

$f_x = 2x + y$

$f_y = x + 2y + 8.$

Notice that $f_x|_P = 0 = f_y|_P$

$f_{xx} = 2$
$f_{yy} = 2$
$f_{xy} = 1$

$D(x,y) = f_{xx} f_{yy} - f_{xy}^2$
$= (2)(2) - (1)^2$
$= 3 > 0.$

$f_{xx}|_P = 2 > 0.$

So $f\left(\frac{8}{3}, \frac{6}{13}\right)$ is local min.