1. Which one of the following equations is an exact differential equation?

(a) $2xy\,dx + (2 + x^2)\,dy = 0$ (correct)

(b) $(x^2 + 1)\,dx - xy\,dy = 0$

(c) $x\,dy + (3x - 2y)\,dx = 0$

(d) $x^2ydy - y\,dx = 0$

(e) $2xy\,dx - (1 + x^2)\,dy = 0$

2. If we solve the differential equation $(\cos x\,\cos y - \cot x)\,dx - \sin x\,\sin y\,dy = 0$, then which one of the following is a solution (here c is a constant)

(a) $\sin x \cos y = \ln(c\,\sin x)$ (correct)

(b) $\sin x \cos y = \ln(c\,\cos x)$

(c) $\sin x \cos y = -\ln(c\,\cos x)$

(d) $\sin x \cos y = -\ln(c\,\sin y)$

(e) $\sin x \cos y = \ln(c\,\tan x)$

3. If we solve the Homogeneous differential equation $(y - \sqrt{x^2 + y^2})\, dx - x\, dy = 0$, then which one of the following is a solution (here c is a constant)

(a) $\sqrt{x^2 + y^2} + y = c$ (correct)

(b) $\sqrt{x^2 + y^2 + y} = c$

(c) $\sqrt{x + y} + y = c$

(d) $\sqrt{x^2 - y} + y = c$

(e) $\sqrt{x^2 + y} + y = c$

4. If we solve the differential equation $\dfrac{dy}{dx} + \dfrac{y}{x} = x^2$, then which one of the following is a solution (here c is a constant)

(a) $xy = \dfrac{x^4}{4} + c$ (correct)

(b) $xy^2 = \dfrac{x^3}{4} + c$

(c) $xy^2 = \dfrac{x^4}{4} + c$

(d) $y = \dfrac{x^3}{4} + c$

(e) $y = \dfrac{x^4}{4} + c$

5. If $y = \sin x$ is an integrating factor of the linear differential equation $\dfrac{dy}{dx} + p(x)y = \sin 2x$, then $p(x)$ can be

(a) $\cot x$ (correct)

(b) $\sin x$

(c) $\ln \sin x$

(d) $\ln \cos x$

(e) $\tan x$

6. If $y = y(x)$ is the solution of the initial value problem $\left(\dfrac{2 + \sin x}{y + 1}\right) \dfrac{dy}{dx} = -\cos x$, $y(0) = 1$, then $y\left(\dfrac{\pi}{2}\right)$ equals

(a) $\dfrac{1}{3}$ (correct)

(b) $\dfrac{-1}{3}$

(c) $\dfrac{-2}{3}$

(d) $\dfrac{2}{3}$

(e) $\dfrac{-4}{3}$

7. The sum of all values of m for which $y = x^m$ is a solution of the differential equation $x^2y'' - 7xy' + 15y = 0$ is

(a) 8 (correct)

(b) -8

(c) 2

(d) -2

(e) 0

8. The sum of all values of c for which $y = c$ is a constant solution of the differential equation $y' = y^2 + 2y - 3$ is

(a) -2 (correct)

(b) 2

(c) -1

(d) 1

(e) 0

9. Using the Existence and Uniqueness Theorem, the initial value problem $(y-x)y' = y+x$, $y(a) = b$ has a unique solution if

(a) $a = 1$, $b = -1$ (correct)

(b) $a = 1$, $b = 1$

(c) $a = 0$, $b = 0$

(d) $a = -1$, $b = -1$

(e) $a = 2$, $b = 2$

10. **(15 points)** By using an appropriate integrating factor, transform the differential equation $(y^2 + xy^3)\,dx + (5y^2 - xy + y^3 \sin y)\,dy = 0$ into an exact equation, then solve it.

11. **(13 points)** Solve the initial value problem $y^{\frac{1}{2}}\dfrac{dy}{dx} + y^{\frac{3}{2}} = 1$, $y(0) = 4$.

12. **(8 points)** Solve the differential equation $\dfrac{dy}{dx} = 2 + \sqrt{y - 2x + 3}$

13. **(10 points)** A small metal bar, whose initial temperature was $20°C$, is dropped into a large container of boiling water. How long will it take the bar to reach $90°C$ if it is known that is temperature increases $2°C$ in one second?
(Note that the temperature of boiling water is $100°c$)