

King Fahd University of Petroleum and Minerals  
Department of Mathematics  
**Math 202**  
**Final Exam**  
**213**  
**August 11, 2022**  
**Net Time Allowed: 180 Minutes**

**USE THIS AS A TEMPLATE**

Write your questions, once you are satisfied upload this file.

1. If the differential equation

$$(6xy^3 + \cos y) dx + (2kx^2y^2 - x \sin y) dy = 0$$

is exact, then  $k$  is equal to

(a)  $\frac{9}{2}$

(b)  $\frac{7}{2}$

(c)  $\frac{11}{2}$

(d)  $\frac{9}{4}$

(e)  $\frac{7}{4}$

2. If  $c$  is a constant, solving  $e^xy \frac{dy}{dx} = e^{-y} + 3e^{-2x-y}$ , gives

(a)  $ye^y - e^y + e^{-x} + e^{-3x} = c$

(b)  $ye^y + e^y + e^{-x} - e^{-3x} = c$

(c)  $ye^y - e^y + e^{-x} + 2e^{-3x} = c$

(d)  $ye^y - e^y + 2e^{-x} - e^{-3x} = c$

(e)  $ye^y + e^y + 2e^{-x} - 2e^{-3x} = c$

3. If  $c$  is a constant, solving  $ydx - 4(x + y^6) dy = 0$ , gives

(a)  $x = 2y^6 + cy^4$

(b)  $x = 2y^4 + cy^2$

(c)  $x = y^6 - cy^4$

(d)  $y = 2x^6 + cx^4$

(e)  $x = 3y^6 + y^4$

4. If  $a, b, c$  and  $d$  are the roots of the auxiliary equation of the differential equation  $y^{(4)} - 7y'' - 18y = 0$ , then  $a + b + c + d =$

(a) 0

(b) 6

(c) -6

(d) 2

(e) -2

5. If  $D^4 + aD^3 + bD^2 + cD + 20$  annihilates  $\frac{2x - 3 \cos x}{e^{2x}}$ , where  $D = \frac{d}{dx}$ , then  $a + b + c =$

- (a) 69
- (b) 68
- (c) 67
- (d) 70
- (e) 71

6. If  $y_p = Ax^2 + Bx$  is a particular solution of the differential equation  $2x^2y'' + 5xy' + y = x^2 - x$ , then  $15A + 6B =$

- (a) 0
- (b) 1
- (c) 2
- (d) -1
- (e) -2

7. The general solution of the differential equation

$$y''' + 2y'' + y' = 10$$

is  $y =$

- (a)  $c_1 + c_2e^{-x} + c_3xe^{-x} + 10x$
- (b)  $c_1 + c_2e^{-x} + 10x$
- (c)  $c_1 + c_2e^{-x} + c_3xe^{-x} + x$
- (d)  $c_1 + c_2e^{-x} + c_3xe^x + 5x$
- (e)  $c_1 + c_2e^{-x} + c_3e^x + 10x$

8. If  $y = y(x)$  is the solution of the initial-value problem  $x^2y'' - xy' + y = 0$ ,  $y(1) = 3$ ,  $y'(1) = -1$ , then  $y(e) =$

- (a)  $-e$
- (b)  $2e$
- (c)  $3e$
- (d)  $-2e$
- (e)  $-3e$

9. The general solution of the differential equation

$$4y'' + 36y = \csc 3x \text{ is } y =$$

- (a)  $c_1 \cos 3x + c_2 \sin 3x - \frac{1}{12}x \cos 3x + \frac{1}{36}(\sin 3x) \ln |\sin 3x|$
- (b)  $c_1 \cos 3x + c_2 \sin 3x + \frac{1}{4}x \sin 3x + \frac{1}{36}(\cos x) \ln |\cos x|$
- (c)  $c_1 \cos 3x + c_2 \sin 3x - x \cos 3x + (\sin 3x) \ln |\sin 3x|$
- (d)  $c_1 \cos 3x + c_2 \sin 3x - x \sin 3x + (\cos x) \ln |\cos x|$
- (e)  $c_1 \cos 3x + c_2 \sin 3x - x \cos 3x + (\cos 3x) \ln |\cos 3x|$

10. If  $c$  is a constant, solving this homogeneous differential equation  $(x-y) dx + xdy = 0$ , gives

- (a)  $y = cx - x \ln |x|$
- (b)  $y = cx^2 - x \ln |x|$
- (c)  $y = cx + x^2 \ln |x|$
- (d)  $y = cx - 2x \ln |x|$
- (e)  $y = cx + 2 \ln |x|$

11. The minimum radius of convergence of the power series solutions for the differential equation  $(x^2 - 2x + 10)y'' + xy' - 4y = 0$  about the ordinary point  $x = 1$ , is  $R =$

- (a) 3
- (b) 2
- (c) 1
- (d) 4
- (e) 5

12. If  $y = \sum_{n=0}^{\infty} c_n x^n$  is a power series solution about the ordinary point  $x_0 = 0$  of the differential equation  $y'' + xy = 0$ , then the coefficients  $c_n$  satisfy

- (a)  $c_{n+2} = \frac{-c_{n-1}}{(n+2)(n+1)}, n \geq 1$
- (b)  $c_{n+2} = \frac{-c_n}{(n+2)(n+1)}, n \geq 1$
- (c)  $c_n = \frac{-c_{n+2}}{(n+2)(n+1)}, n \geq 1$
- (d)  $c_{n+2} = \frac{-1}{(n+2)(n+1)}, n \geq 1$
- (e)  $c_{n+2} = \frac{-c_{n-1}}{(n+1)(n+3)}, n \geq 1$

13. If we solve the differential equation  $3xy'' + y' - y = 0$  about the regular singular point  $x = 0$ , by considering  $y = \sum_{n=0}^{\infty} c_n x^{n+r}$  then we will have the recurrence relation

$$c_{k+1} = \frac{c_k}{(k+r+1)(3k+3r+1)}, \quad k \geq 0$$

where  $r$  are the roots of the indicial equation. The first three nonzero terms in the series solution that corresponds to the smaller indicial root evaluated at  $x = 1$  could be

- (a)  $1, 1, \frac{1}{8}$
- (b)  $2, 3, \frac{1}{6}$
- (c)  $2, 2, \frac{2}{5}$
- (d)  $1, 1, \frac{1}{6}$
- (e)  $2, 2, \frac{1}{8}$

14. Which one of the following statements is TRUE about the differential equation

$$(x^3 - 2x^2 - 3x)^2 y'' + x(x-3)^2 y' - (x+1)y = 0?$$

- (a)  $x = -1$  is an irregular singular point
- (b)  $x = 1$  is an irregular singular point
- (c)  $x = 0$  is an irregular singular point
- (d)  $x = 3$  is an irregular singular point
- (e)  $x = 3$  is an ordinary point



15. If  $X = \begin{pmatrix} 1 \\ b \end{pmatrix} e^{-5t}$  is a solution of the system

$$\frac{dx}{dt} = 3x - 4y,$$

$$\frac{dy}{dt} = 4x - 7y,$$

then  $b =$

- (a) 2
- (b) 0
- (c) -1
- (d) -2
- (e) 1

16. If the general solution of the system

$$\frac{dx}{dt} = x + y - z$$

$$\frac{dy}{dt} = 2y$$

$$\frac{dz}{dt} = y - z$$

is given by

$$X = c_1 \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} e^{\lambda_1 t} + c_2 \begin{pmatrix} c \\ d \\ 1 \end{pmatrix} e^{\lambda_2 t} + c_3 \begin{pmatrix} e \\ f \\ 2 \end{pmatrix} e^{-t}, \lambda_1 \neq \lambda_2$$

Then  $a \cdot b + c \cdot d + e \cdot f =$

- (a) 6
- (b) 5
- (c) 4
- (d) 7
- (e) 8

17. Consider the following initial value problem

$$X' = AX, \quad X(0) = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

where  $A = \begin{bmatrix} 2 & 4 \\ -1 & 6 \end{bmatrix}$ . Then  $X(1) =$

(a)  $\begin{bmatrix} 25e^4 \\ 19e^4 \end{bmatrix}$

(b)  $\begin{bmatrix} 19e^4 \\ 25e^4 \end{bmatrix}$

(c)  $\begin{bmatrix} 10e^4 \\ 15e^4 \end{bmatrix}$

(d)  $\begin{bmatrix} 25e^4 \\ 10e^4 \end{bmatrix}$

(e)  $\begin{bmatrix} 15e^4 \\ 19e^4 \end{bmatrix}$

18. The solution of  $X' = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} X$ ,  $X(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$  at  $t = \frac{\pi}{4}$  equals

(a)  $\begin{bmatrix} -2 \\ 1 \end{bmatrix} e^{\frac{\pi}{4}}$

(b)  $\begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{\frac{\pi}{4}}$

(c)  $\begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{\frac{\pi}{4}}$

(d)  $\begin{bmatrix} -2 \\ 0 \end{bmatrix} e^{\frac{\pi}{4}}$

(e)  $\begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-\frac{\pi}{4}}$

19. Consider the non homogeneous system

$$X' = AX + \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^t$$

then the particular solution  $X_p(-1) =$

(a)  $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$

(b)  $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

(c)  $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

(d)  $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

(e)  $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

20. If the exponential matrix of

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{pmatrix}$$

is given by

$$e^{At} = \begin{pmatrix} t+1 & t & t \\ h_1(t) & t+1 & t \\ h_2(t) & h_3(t) & -2t+1 \end{pmatrix}$$

then  $h_1(1) + h_2(2) + h_3(3) =$

(a)  $-9$

(b)  $9$

(c)  $-10$

(d)  $10$

(e)  $-8$

21. The first order differential equation

$$\frac{dr}{d\theta} = r\theta + r + \theta + 1$$

is not separable

(Answer True or False by filling in the OMR sheet)

(a) False

(b) True

22. If  $y_1 = e^x$  and  $y_2 = e^{-x}$  are solutions of a homogeneous linear differential equation (DE), then  $y = -5e^{-x} + 10e^x$  is also a solution of the DE.

(Answer True or False by filling in the OMR sheet)

(a) True

(b) False

23. The linear differential equation,  $y' + k_1y = k_2$ , where  $k_1$  and  $k_2$  are non zero constants, possesses a constant solution  
(Answer True or False by filling in the OMR sheet)

(a) True

(b) False

24. If the set consisting of two functions  $f_1$  and  $f_2$  is linearly independent on an interval  $I$ , then the Wronskian  $W(f_1, f_2) \neq 0$  for all  $x$  in  $I$ .  
(Answer True or False by filling in the OMR sheet)

(a) False

(b) True

25.  $k_1 = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$  is an eigenvector of the coefficient matrix

$$\begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{pmatrix}.$$

(Answer True or False by filling in the OMR sheet)

- (a) True
- (b) False