

1. The **sum** of all constant solutions of the differential equation

$$\frac{dy}{dx} = y(y^2 - 1)$$

is

- (a) 0
- (b) 3
- (c) 2
- (d) 4
- (e) -1

(correct)

2. The **sum** of all values of  $c$  such that  $y = \frac{c}{1+x^2}$  is a solution of the differential equation  $\frac{dy}{dx} = 2xy^2$  is

- (a) -1
- (b) 1
- (c) 0
- (d) 2
- (e) 3

(correct)

3. An explicit particular solution of the initial value problem

$$\frac{dy}{dx} = 3x^2(y^2 + 1), \quad y(0) = 1$$

is

- (a)  $y = \tan\left(x^3 + \frac{\pi}{4}\right)$  (correct)
- (b)  $y = \tan\left(2x^3 + \frac{\pi}{4}\right)$
- (c)  $y = \tan\left(3x^3 + \frac{\pi}{4}\right)$
- (d)  $y = \tan\left(4x^3 + \frac{\pi}{4}\right)$
- (e)  $y = \tan\left(x^3 - \frac{\pi}{4}\right)$

4. In a certain culture of bacteria, the initial population was 5, and it increased to be 40 in 15 hours. How long did it take for the population to be 10?

- (a) 5 hours (correct)
- (b) 4 hours
- (c) 3 hours
- (d) 2 hours
- (e) 1 hour

5. The matrix  $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 4 \\ 4 & k & 2 \end{bmatrix}$  is invertible if  $k \neq$

- (a) 3
- (b) 2
- (c) 0
- (d) -6
- (e) 1

(correct)

6. The values of  $a$  and  $b$  that make the differential equation

$$(ye^{ay} + bxy) dx + ((1 + 2y)e^{ay}x + 3x^2) dy = 0$$

exact are:

- (a)  $a = 2, b = 6$
- (b)  $a = 2, b = 3$
- (c)  $a = 2, b = 2$
- (d)  $a = 6, b = 3$
- (e)  $a = 3, b = 6$

(correct)

7. (10 points) Given the system

$$\begin{cases} x + 2y = 3 \\ 2x - y + z = -1 \\ 4x - y + 2z = 2 \end{cases}$$

Use Cramer's Rule to find the value of  $x$ .

$$\begin{aligned} \begin{vmatrix} 1 & 2 & 0 \\ 2 & -1 & 1 \\ 4 & -1 & 2 \end{vmatrix} &= 1 \begin{vmatrix} -1 & 1 \\ -1 & 2 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix} \\ &= (-2+1) - 2(4-4) \quad (4 \text{ pts}) \\ &= -1 \end{aligned}$$

$$\begin{aligned} \begin{vmatrix} 3 & 2 & 0 \\ -1 & -1 & 1 \\ 2 & -1 & 2 \end{vmatrix} &= 3 \begin{vmatrix} -1 & 1 \\ -1 & 2 \end{vmatrix} - 2 \begin{vmatrix} -1 & 1 \\ 2 & 2 \end{vmatrix} \\ &= 3(-2+1) - 2(-2-2) \quad (4 \text{ pts}) \\ &= -3+8=5 \end{aligned}$$

$$x = \frac{\begin{vmatrix} 3 & 2 & 0 \\ -1 & -1 & 1 \\ 2 & -1 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 0 \\ 2 & -1 & 1 \\ 4 & -1 & 2 \end{vmatrix}} = \frac{5}{-1} = -5 \quad (2 \text{ pts})$$

8. (10 points) Solve the initial value problem

$$xy' - 3y = x^4 \cos x, \quad y(2\pi) = 0.$$

The DE is linear. We put it in standard form

$$y' - \frac{3}{x}y = x^3 \cos x \quad \dots (1) \quad (1 \text{ pt})$$

$$\text{integrating factor } u(x) = e^{\int -\frac{3}{x} dx} = e^{-3 \ln x} = \frac{1}{x^3}, \quad x > 0 \quad (2 \text{ pts})$$

We multiply Eq(1) by  $\frac{1}{x^3}$  to get

$$\frac{d}{dx} \left( \frac{1}{x^3} y \right) = \cos x \quad (2 \text{ pts})$$

$$\Rightarrow \int \frac{d}{dx} \left( \frac{y}{x^3} \right) dx = \int \cos x dx$$

$$\Rightarrow \frac{y}{x^3} = \sin x + C \quad \text{or } y = x^3 \sin x + Cx^3$$

$$y(2\pi) = 0 \Rightarrow C = 0. \quad (1 \text{ pt})$$

The solution of the IVP is

$$y = x^3 \sin x \quad (1 \text{ pt})$$

9. (12 points) Let  $A = \begin{bmatrix} 2 & 4 & 2 \\ 4 & 11 & 8 \\ 2 & 5 & 4 \end{bmatrix}$ .

Find the inverse of  $A$  using the adjoint formula.

$$|A| = \begin{vmatrix} 2 & 4 & 2 \\ 4 & 11 & 8 \\ 2 & 5 & 4 \end{vmatrix} = 2 \begin{vmatrix} 11 & 8 \\ 5 & 4 \end{vmatrix} - 4 \begin{vmatrix} 4 & 8 \\ 2 & 4 \end{vmatrix} + 2 \begin{vmatrix} 4 & 11 \\ 2 & 5 \end{vmatrix}$$

$$= 2(44 - 40) - 4(16 - 16) + 2(20 - 22) = 8 - 4 = 4 \quad (2 \text{ pts})$$

$$A_{11} = \begin{vmatrix} 11 & 8 \\ 5 & 4 \end{vmatrix} = 4, \quad A_{12} = - \begin{vmatrix} 4 & 8 \\ 2 & 4 \end{vmatrix} = 0, \quad A_{13} = \begin{vmatrix} 4 & 11 \\ 2 & 5 \end{vmatrix} = -2$$

$$A_{21} = - \begin{vmatrix} 4 & 2 \\ 5 & 4 \end{vmatrix} = -6, \quad A_{22} = \begin{vmatrix} 2 & 2 \\ 2 & 4 \end{vmatrix} = 4, \quad A_{23} = - \begin{vmatrix} 2 & 4 \\ 2 & 5 \end{vmatrix} = -2$$

$$A_{31} = \begin{vmatrix} 4 & 2 \\ 11 & 8 \end{vmatrix} = 10, \quad A_{32} = - \begin{vmatrix} 2 & 2 \\ 4 & 8 \end{vmatrix} = -8, \quad A_{33} = \begin{vmatrix} 2 & 4 \\ 4 & 11 \end{vmatrix} = 6$$

$$[A_{ij}] = \begin{bmatrix} 4 & 0 & -2 \\ -6 & 4 & -2 \\ 10 & -8 & 6 \end{bmatrix} \Rightarrow [A_{ij}]^T = \begin{bmatrix} 4 & -6 & 10 \\ 0 & 4 & -8 \\ -2 & -2 & 6 \end{bmatrix} \quad (8 \text{ pts})$$

$$\bar{A} = \frac{[A_{ij}]^T}{|A|} = \begin{bmatrix} 1 & -\frac{3}{2} & \frac{5}{2} \\ 0 & 1 & -2 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{3}{2} \end{bmatrix} \quad (2 \text{ pts})$$