1. If $y_p = Ax + B$ is a particular solution of the differential equation

$$y'' - y' - 2y = 2x,$$

then $A + B =$

(a) $-\frac{1}{2}$ (correct)

(b) $\frac{1}{2}$

(c) $\frac{3}{2}$

(d) 2

(e) 0

$y_p' = A$, $y_p'' = 0$

$y_p'' - y_p' - 2y_p = 2x$

$0 - A - 2(Ax+B) = 2x$

$-2Ax - A - 2B = 2x$

$\Rightarrow \begin{cases} -2A = 2 \\ -A - 2B = 0 \end{cases} \Rightarrow A = -1, \; B = +\frac{1}{2}$

$A + B = -1 + \frac{1}{2} = -\frac{1}{2}$

2. The general solution of the ODE

$$2y''' - 3y'' - 2y' = 0$$

is

(a) $y = c_1 + c_2 e^{2x} + c_3 e^{-x/2}$ (correct)

(b) $y = c_1 + c_2 e^{-2x} + c_3 e^{x/2}$

(c) $y = c_1 + c_2 e^{-x} + c_3 e^{-2x}$

(d) $y = c_1 e^{2x} + c_2 e^{-x/2}$

(e) $y = c_1 + c_2 e^{2x} + c_3 x e^{2x}$

$2r^3 - 3r^2 - 2r = 0$

$r(2r^2 - 3r - 2) = 0$

$r(2r+1)(r-2) = 0$

$r = 0, -\frac{1}{2}, 2$

$y = c_1 e^{0x} + c_2 e^{2x} + c_3 e^{-\frac{1}{2}x}$

$= c_1 + c_2 e^{2x} + c_3 e^{-x/2}$

3. Which set of the following functions is linearly independent on $(-\infty, \infty)$.

(a) $f(x) = 2,\ g(x) = x,\ h(x) = x^2$ no one is a linear combination of the others (correct)

(b) $f(x) = 1,\ g(x) = 2 - x^2,\ h(x) = 3 + x^2$ $\to f(x) = \frac{1}{5} g(x) + \frac{1}{5} h(x)$

(c) $f(x) = 5,\ g(x) = \sin^2 x,\ h(x) = \cos(2x)$

(d) $f(x) = 0,\ g(x) = \cos x,\ h(x) = e^{-x}$ $f(x) = 0 \cdot g(x) + 0 \cdot h(x)$

(e) $f(x) = x,\ g(x) = \sin(2x),\ h(x) = \sin x \cos x$ $g(x) = 2h(x) + 0 \cdot f(x)$

$h(x) = \frac{1}{5} f(x) - 2g(x)$

4. An appropriate form of a particular solution y_p for the differential equation

$$y^{(5)} - y^{(3)} = -3x^2 + 5 + 2e^x$$

is

(a) $y_p = Ax^3 + Bx^4 + Cx^5 + Dxe^x$ (correct)

(b) $y_p = A + Bx + Cx^2 + De^x$

(c) $y_p = A + Bx + Cx^2 + Dxe^x$

(d) $y_p = Ax + Bx^2 + Cx^3 + Dxe^x$

(e) $y_p = Ax^2 + Bx^3 + Cx^4 + Dx^2e^x$

$y^{(5)} - y^{(3)} = 0 \Rightarrow r^5 - r^3 = 0 \Rightarrow r^3(r^2 - 1) = 0 \Rightarrow r = 0$ (3 times), $r = -1$, $r = 1$

$y_c = C_1 + C_2 x + C_3 x^2 + C_4 e^{-x} + C_5 e^x$

there is a duplicate with y_c

$g(x) = -3x^2 + 5 + 2e^x \Longrightarrow y_p = \underbrace{A + Bx + Cx^2}_{\cdot x^3} + \underbrace{De^x}_{\cdot x}$

$\Rightarrow y_p = Ax^3 + Bx^4 + Cx^5 + Dxe^x$, no duplicate with y_c

5. A linear homogeneous differential equation whose general solution is

$$y = (A + Bx + Cx^2)\, e^{2x}$$

$r = 2$ repeated 3 times

is given by

$(r-2)^3 = 0$

$r^3 - 3r^2(2) + 3r \cdot 2^2 - 2^3 = 0$

$r^3 - 6r^2 + 12r - 8 = 0$

$y''' - 6y'' + 12y' - 8y = 0$

(a) $y''' - 6y'' + 12y' - 8y = 0$ (correct)

(b) $y''' - 6y'' + 10y' - 8y = 0$

(c) $y'' - 6y' - 8y = 0$

(d) $y''' + 6y'' + 6y' + 8y = 0$

(e) $y''' - 8y = 0$

6. Let A be a 5×8 matrix with real entries. If the rank of A is 3, then the dimension of the solution space of the system $A\mathbf{X} = 0$ is

$\underbrace{\#\text{ of Leading Var.}}_{=\text{rank of } A = 3} + \underbrace{\#\text{ of Free Var.}}_{=\text{dim of Sol. space}} = 8$

(a) 5 (correct)

(b) 4

(c) 3

(d) 2

(e) 8

So dim of Sol. space $= 8 - 3 = 5$

7. Let $u = (1, 1, 0)$, $v = (0, 1, 1)$, $w = (1, 2, -2)$ be vectors in $\mathbb{R}^3$. If

$$(2, 3, 4) = au + bv + cw,$$

then $a^2 + b^2 + c^2 =$

$$\begin{cases} a \quad\quad + c = 2 \\ a + b + 2c = 3 \\ \quad\ b - 2c = 4 \end{cases}$$

(a) 14

(b) 12

(c) 9

(d) 16

(e) 20

Solving the system, we get

$a = 3$, $b = 2$, $c = -1$

So $a^2 + b^2 + c^2 = 9 + 4 + 1 = 14$

(correct)

8. Find **all** real values of k so that the vectors

$$u = (1, 2, 4),\ v = (2, -1, k),\ w = (-1, 4, 2)$$

form a basis for $\mathbb{R}^3$. The three vectors form a basis for $\mathbb{R}^3$ if and only if they L. indep & they are L. indep if and only

(a) $k \neq 3$

(b) $k \neq 0$

(c) $k \neq 1$

(d) $k \neq 2$

(e) $k \neq -5$

$$\begin{array}{ccc} u & v & w \end{array}$$
$$\begin{vmatrix} 1 & 2 & -1 \\ 2 & -1 & 4 \\ 4 & k & 2 \end{vmatrix} \neq 0$$

(correct)

$1(-2-4k) - 2(4-16) - (2k+4) \neq 0$

$-2 - 4k + 24 - 2k - 4 \neq 0$

$\Rightarrow \quad -6k + 18 \neq 0$

$\Rightarrow \quad k \neq 3$

9. The dimension of the subspace

$$W = \{(x, y, z, w) : x - y + 2z - w = 0\}$$

of $\mathbb{R}^4$ is

= Solution space of the System

$x - y + 2z - w = 0$

$[1 \ -1 \ 2 \ -1 \ \vdots \ 0]$

Lead Var: x

Free Var: $y, z, w \Rightarrow \dim(W) = 3$

(a) 3 (correct)

(b) 2

(c) 1

(d) 4

(e) 0

10. Which one of the following statements is **TRUE** about the following subsets of $\mathbb{R}^3$

$U = \{(x, y, z) : x + y = z\}$ ✓ check the two conditions ✓

$V = \{(x, y, z) : x + y = z + 1\}$ $(0,0,0) \notin V$. So V is not a subspace

$W = \{(x, y, z) : x^2 + y^2 = z^2\}$ $(1,0,1), (0,1,1) \in W$

But $(1,0,1) + (0,1,1) = (1,1,2) \notin W$

So W is not a subspace

(a) U is a subspace of $\mathbb{R}^3$ (correct)

(b) V is a subspace of $\mathbb{R}^3$

(c) W is a subspace of $\mathbb{R}^3$

(d) U and W are subspaces of $\mathbb{R}^3$

(e) none of U, V, W is a subspace of $\mathbb{R}^3$