KFUPM/ Department of Mathematics & Statistics/201/MATH 210/ MakeUp Exam

Name: **ID #:** **Serial #:**

Please write clearly and justify all your answers.

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1. [12pts] (a) Let a be an odd integer and $b \in \mathbb{N}$. Prove that either $a \nmid b$ or $a \nmid (b+4)$

(b) Prove that $\sqrt{10}$ is irrational.

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2. [12pts] (a) Use induction to show that for each $n \in \mathbb{N}$,

$$1 + 4 + 7 + \cdots + (3n-2) = \frac{n(3n-1)}{2}$$

(b) Prove that for each $n \in \mathbb{N}$, $3^n \geq n + 2^n$

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3. [12pts] (a) Let R be the relation on $\mathbb{Q}$ given by aRb iff $\sqrt{ab} \in \mathbb{Q}$. Determine whether R is (i) reflexive, (ii) symmetric,(iii) transitive. Is R an equivalence relation?

(b) Find an integer r such that $0 \leq r \leq 10$ and $[80^{12}] + [12^{80}] = [r]$ in $\mathbb{Z}_{11}$.

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4. [12pts] (a) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be the function given by $f(x) = 1 + (1 + x^3)^{1/5}$ for each x in $\mathbb{R}$. Prove that f a bijection and find its inverse function f^{-1}.

(b) Let $g : \mathbb{Z} \longrightarrow \mathbb{Z} \times \mathbb{N}$ be the function given by $g(k) = (k^2, |k| + 1)$ for each k in $\mathbb{Z}$.

(i) Is g one-to-one? Is it onto? Justify.

(ii) Find $(m, n) \in \mathbb{Z} \times \mathbb{N}$ such that $2 \in g^{-1}(\{(m, n)\})$