King Fahd University of Petroleum & Minerals Department of Mathematics

Math 210 Introduction to Sets and Structures (Term 212)

MIDTERM EXAM (Duration = 100 minutes)

Exercise 1

Consider the following collection of sets

$$\{[2,2+1),[2,2+\frac{1}{2}),[2,2+\frac{1}{3}),\ldots\}$$

Define a set A_n , for each $n \in \mathbb{N}$, such that the indexed collection $\{A_n\}_{n \in \mathbb{N}}$ is precisely the above collection of sets; and then find $\bigcup_{n \in \mathbb{N}} A_n$ and $\bigcap_{n \in \mathbb{N}} A_n$.

Exercise 2

Consider the open sentences

$$P(x): x = -2$$
 and $Q(x): x^2 = 4$

over the domain $S = \{-2, 0, 2\}$. Determine all $x \in S$ for which the following statements are **TRUE**.

- (a) $\sim P(x)$
- (b) $P(x) \vee Q(x)$
- (c) $P(x) \wedge Q(x)$
- (d) $P(x) \Rightarrow Q(x)$
- (e) $Q(x) \Rightarrow P(x)$
- (f) $P(x) \Leftrightarrow Q(x)$

Exercise 3

Prove for every integer $n \ge 8$ that there exist nonnegative integers a and b such that n = 3a + 5b.

Exercise 4

Let n be an integer ≥ 2 and let $x_1, x_2, ..., x_n$ be positive real numbers. Prove that

$$n\sum_{i=1}^{n} x_i^2 \ge \sum_{1 \le i < j \le n} 2x_i x_j + \sum_{i=1}^{n} x_i^2$$

Exercise 5

Use proof by minimum counterexample to prove that $6 \mid 7n(n^2 - 1)$ for every positive integer n.

Exercise 6

Prove or disprove: There exist equivalence relations R_1 and R_2 on the set $S = \{a, b, c\}$ such that $R_1 \not\subseteq R_2$, $R_2 \not\subseteq R_1$ and $R_1 \cup R_2 = S \times S$.

Exercise 7

A relation R is defined on **Z** by a R b if $2a + 2b \equiv 0 \pmod{4}$. Prove that R is an equivalence relation and determine the distinct equivalence classes.

Exercise 8

Prove that the function $f: R - \{1\} \to R - \{3\}$ defined by $f(x) = \frac{3x+1}{x-1}$ is bijective.

Exercise 9

Two functions $f: \mathbf{Z}_{10} \to \mathbf{Z}_{10}$ and $g: \mathbf{Z}_{10} \to \mathbf{Z}_{10}$ are defined by f([a]) = [3a] and g([a]) = [7a].

- (a) Determine $g \circ f$ and $f \circ g$.
- (b) What can be concluded as a result of (a)?