KFUPM/ Department of Mathematics/T221/MATH 210/Exam 1

Name: ID#:

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1. [8pts] Let P, Q, R be statements. Are the statements $(P \longrightarrow R) \vee Q$ and $(\sim P \wedge Q) \longrightarrow R$ logically equivalent? Justify.

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2. [6pts] Mark each of the following statements as TRUE or FALSE and justify your choice.

(a) $\forall x \in \mathbb{Q}, \exists y \in \mathbb{N}, xy \geq 0$.

(b) $\exists x \in \mathbb{Q}, \forall y \in \mathbb{N}, xy \geq x + 7$.

(c) $\forall x \in \mathbb{R}, \exists y \in \mathbb{N}, xy = x$.

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3. [6pts] Let $S_i = \{i, 2i+1\}$ $(i \in \mathbb{N})$.

(a) Find $\bigcup_{i=1}^{4} S_i$.

(b) Find $|\mathcal{P}(S_{10} \times S_{21})|$.

(c) Find all positive integers i such that $|S_5 \cup S_i| = 3$.

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4. [10pts] (a) Let A, B be subsets of some universal set U. Prove that $\overline{A} \subseteq \overline{B}$ if and only if $B \subseteq A$.

(b) Prove that if A and B are nonempty sets such that $A \times B \subseteq B \times A$, then $A = B$.

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5. [5pts] Let $x, y \in \mathbb{R}$. Prove that $x^2 + y^2 \geq 2|xy|$.

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6. [10pts] (a) Let $a, m \in \mathbb{Z}$, where $m|a$. Prove that if a is odd, then m is odd.

(b) Let $b \in \mathbb{Z}$. Prove that $b^3 \equiv b \pmod{3}$. Is it true that $b^5 \equiv b \pmod{3}$? Justify.