King Fahd University of Petroleum & Minerals
Department of Mathematics

Math 225 Introduction to Linear Algebra (Term 221)
Test 1 (Duration = 50 minutes)

Exercise 1

For each of the following systems, provide the augmented matrix, the reduced row echelon form, the reduced system, and then the solutions.

(a)
$$\begin{aligned} x_1 + x_2 &= 1 \\ x_1 - x_2 &= 3 \\ -x_1 + 2x_2 &= -2 \end{aligned}$$

(b)
$$\begin{aligned} x_1 + x_2 + x_3 + x_4 + x_5 &= 2 \\ x_1 + x_2 + x_3 + 2x_4 + 2x_5 &= 3 \\ x_1 + x_2 + x_3 + 2x_4 + 3x_5 &= 2 \end{aligned}$$

Exercise 2

Compute the LU factorization of $\begin{pmatrix} 1 & 1 & 1 \\ 3 & 5 & 6 \\ -2 & 2 & 7 \end{pmatrix}$.

Exercise 3

Let $A = \begin{pmatrix} 2 & 1 & 2 \\ 0 & 3 & 4 \\ 1 & 1 & 2 \end{pmatrix}$ and set $A^{-1} = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}$.

Compute $x_{11}, x_{32}, x_{23}$by using Cramer's rule.

Exercise 4

Let A and B be two *symmetric* $n \times n$ matrices. Prove: $AB = BA \Leftrightarrow AB\ is\ sysmmetric$.

Exercise 5

Let $A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}$; $B = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}$; $E = \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}$

Show that if $A = EB$, then $det(A + B) = det(A) + det(B)$.

King Fahd University of Petroleum & Minerals
Department of Mathematics

Math 225 Introduction to Linear Algebra (Term 221)
Test 2 (Duration = 50 minutes)

Exercise 1

Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$; $v_2 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$; $v = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$. Does $v \in Span(v_1, v_2)$?

Exercise 2

Are the following polynomials Linearly Independent in P_4 :

$$\begin{aligned} p_1 &= 1 - x + 2x^2 + 3x^3 \\ p_2 &= -2 + 3x + x^2 - 2x^3 \\ p_3 &= 1 + 7x^2 + 7x^3 \end{aligned}$$

Exercise 3

Let S be the subspace of P_3 consisting of all polynomials $p(x)$ such that $p(1) = 0$, and let T be the subspace of all polynomials $q(x)$ such that $q(-1) = 0$. Find bases for:

(a) S (b) T (c) $S \cap T$

Exercise 4

Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$; $v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$; $v_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $u_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$; $u_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$; $u_3 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$.

(a) Find the transition matrix from the basis $E = \{v_1, v_2, v_3\}$ to the basis $F = \{u_1, u_2, u_3\}$
(b) If x $= -v_2 + 2v_3$, determine $[\text{x}]_F$

Exercise 5

Let $A = \begin{pmatrix} 1 & 2 & 2 & 3 & 1 & 4 \\ 2 & 4 & 5 & 5 & 4 & 9 \\ 3 & 6 & 7 & 8 & 5 & 9 \end{pmatrix}$.

(a) Find the *rank* of A.
(b) Find a basis for the *column space* of A.
(c) Find a basis for the *null space* of A.