

**King Fahd University of Petroleum & Minerals**  
**Department of Mathematics & Statistics**  
**Math 333 Major Exam 2**  
**The First Semester of 2021-2022 (211)**

**Time Allowed: 120 Minutes**

---

Name: \_\_\_\_\_ ID#: \_\_\_\_\_

Instructor: \_\_\_\_\_ Sec #: \_\_\_\_\_ Serial #: \_\_\_\_\_

---

- Mobiles and calculators are not allowed in this exam.
  - Write all steps clear.
- 

Question #	Marks	Maximum Marks
1		10
2		12
3		10
4		12
5		13
6		10
7		8
Total		75

**Q:1** (5+5 points) Find the Laplace transform of the following:

(a)  $f(t) = t e^{3t} \sin^2 t$

(b)  $f(t) = e^{t-2} \sin(2t - 4) \mathcal{U}(t - 2)$

**Q:2** (12 points) Solve the initial value problem using Laplace transform

$$y'' + 3y' + 2y = f(t) \text{ with } y(0) = 0, y'(0) = 0, \text{ where } f(t) = \begin{cases} 1 & 0 \leq t < 3 \\ -1 & t \geq 3 \end{cases}.$$

**Q:3** (10 points) Find  $y(t)$  by solving the integral equation using the Laplace transform

$$y'(t) + 9 \int_0^t y(\tau) d\tau = \sin(3t), \quad y(0) = 0$$

**Q:4** (10+2 points) Find the Fourier series of the function  $f(x) = \begin{cases} 2+x & -2 < x < 0 \\ 2 & 0 \leq x < 2 \end{cases}$ .

Use the Fourier series to show that  $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$



**Q:5** (10+3 points) (a) Find eigenvalues and eigenfunctions of the boundary value problem

$$x^2y'' + xy' + \lambda y = 0, \quad y(1) = 0, \quad y(e) = 0.$$

(b) Put the differential equation in self-adjoint form and write orthogonality relation.





**Q:6** (10 points) Expand  $f(x) = 1$ ,  $0 < x < 3$  into Fourier Bessel series using the Bessel functions of order ZERO under the boundary condition  $J_0(3\alpha) + \alpha J_0'(3\alpha) = 0$ .

**Q:7** (8 points) Expand  $f(x) = x^2$ ,  $-1 < x < 1$  into Fourier Legendre series. Find only first TWO non-zero terms of the series.