King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 333 Final Exam
The First Semester of 2021-2022 (211)

<u>Time Allowed</u>: 150 Minutes

Name: ______________________ ID#: ______________

Instructor: ____________________ Sec #: ________ Serial #: ________

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		20
2		20
3		18
4		18
5		15
6		14
Total		105

Q:1 (20 points) Use separation of variables method to solve the initial-boundary value problem:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < L, \; t > 0,$$

with boundary conditions

$$u_x(0,t) = 0, \quad u_x(L,t) = 0, \quad t > 0,$$

and initial condition

$$u(x,0) = x, 0 < x < L.$$

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Q:2 (20 points) Use separation of variables method to solve the Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < \pi, \; 0 < y < \pi,$$

subject to the boundary conditions

$$\begin{aligned} u(0, y) &= 0, \quad u(\pi, y) = 0, \quad 0 < y < \pi, \\ u(x, 0) &= 0, \quad u(x, \pi) = 1, \quad 0 < x < \pi. \end{aligned}$$

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Q:3 (18 points) Use separation of variables method to find the steady state temperature $u(x, z)$

in a right circular cylinder by solving the problem

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} = 0,\ 0 < r < 2,\ 0 < z < 4,$$

subject to the boundary conditions

$$\begin{aligned} u\,(2, z) &= 0,\ 0 < z < 4, \\ u\,(r, 0) &= 0,\ 0 < r < 2, \\ u\,(r, 4) &= 4,\ 0 < r < 2. \end{aligned}$$

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Q:4 (18 points) Use separation of variables method to find the steady-state temperature $u(r, \theta)$

in a sphere of radius 2 by solving the problem

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial u}{\partial \theta} = 0, \quad 0 < r < 2, \ 0 < \theta < \pi,$$

subject to the boundary condition

$$u(2, \theta) \quad = \quad 1 - 2\cos(\theta), \quad 0 < \theta < \pi.$$

Write first two nonzero terms of the series solution.

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Q:5 (15 points) Use Laplace transform to solve the problem

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \ x > 0, \ t > 0$$

subject to the conditions

$$u(0,t) = \sin(2t), \quad \lim_{x \to \infty} u(x,t) = 0, \ t > 0,$$

$$u(x,0) = e^{-3x}, \quad u_t(x,0) = 0, \ x > 0.$$

Q:6 (14 points) Use appropriate Fourier transform to solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < 4, \ y > 0,$$

subject to the conditions

$$\begin{aligned} u(0,y) &= 0, \ \ u(4,y) = \begin{cases} 2 & 0 < y < 2 \\ 0 & y > 2 \end{cases}, \\ \left.\frac{\partial u}{\partial y}\right|_{y=0} &= 0, \quad 0 < x < 4. \end{aligned}$$