

**King Fahd University of Petroleum and Minerals****Mathematics Department****Math 371 Exam 2, 1<sup>st</sup> Semester (221),****Net Time Allowed: 90 minutes****November 9, 2022****Name:****ID No.:****Section NO.:**Please:

1. Write clearly with a **pen or dark pencil** in the **designed area for each question**.
2. **Fill your info clearly**, and write your **ID NO** in each paper (pages 2, 4, 6, 8, 10) in the right corner **inside the box**.
3. **If you need more space**, you may use page 10 and 11 but state that clearly in the question's area.
4. Show **all** your steps, no credit will be given to wrong steps.
5. Set your calculator to RADIAN

1) Consider the function  $f(x) = x^2 \ln x + 5$ .

- a. Approximate  $f'(1.5)$  using **three-point midpoint** formula with  $h = 0.05$ .
- b. Compute the **error bound** for  $f'(1.5)$  using the formula used in a.

11 points

- 2) Determine the values of  $n$  and  $h$  required to approximate  $\int_0^1 e^x \sin x \, dx$  within  $10^{-3}$  using the **Composite Trapezoidal rule**.

8 points

3) Show that the initial value problem

$$\begin{aligned} e^{-t} \frac{dy}{dt} &= te^{-y}, & 0 \leq t \leq 2, \\ y(0) &= 5, & 0 \leq y \leq 1, \end{aligned}$$

7 points

has a **unique solution**. (Do not solve the IVP)

4) Consider the initial value problem

$$\frac{dy}{dt} - y = e^{-t}, \quad 0 \leq t \leq 2,$$
$$y(0) = 1,$$

10 points

- a. Use the **Euler's method** with step size  $h = 0.2$  to approximate  $y(0.8)$ . (Use 4 decimal places for all calculations.)
- b. Given that  $|y''(t)| \leq \frac{e^2(3-e^{-4})}{2}$  on  $[0, 2]$ , find an **error bound** for the approximation of  $y(0.8)$  in part a.

- 5) The **Trapezoidal rule** applied to  $\int_{-5}^1 f(x) dx$  gives the value 6 and **Simpson's rule** gives the value 3. What is  $f(-2)$ ?

8 points

- 6) A car traveling along a straight road is clocked at a number of points. The data from the observations are given in the following table, where the time is in seconds, the distance is in feet

Time	0	3	5	8	<b>10</b>	13
Distance	0	225	383	623	742	993

8 points

**Approximate** the speed and the acceleration of the car at  $t = 10$  s.

- 7) Find the least squares **polynomial of degree two** that approximates the data below which represents the bacterial growth in a liquid culture over a number of days. Given that, the **coefficient of  $x^2$**  in the polynomial is  $\frac{1}{6}$ . (Show **all** necessary work, do not use a built-in command in the calculator)

Day	0	2	3	5
Amount	2	10	15	25

12 points



- 8) Construct the **clamped cubic spline**  $S(x)$  that passes through the points  $(1,1)$ ,  $(2,4)$ , and  $(3,9)$ , where  $S'(1) = 2$  and  $S'(3) = 6$ .

12 points

\*Please, you need to write in the bottom of the question where you need more space "**Go to page 10**".

\*Please, you need to write in the bottom of the question where you need more space **“Go to page 11”**.

\*Please, you need to write in the bottom of the question where you need more space "**Go to page 12**".