

**King Fahd University of Petroleum & Minerals**  
**Department of Mathematics & Statistics**  
**Math 445 Major Exam I**  
**The Second Semester of 2021-2022 (212)**

**Time Allowed: 90 Minutes**

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Name: \_\_\_\_\_ ID#: \_\_\_\_\_

Section/Instructor: \_\_\_\_\_ Serial #: \_\_\_\_\_

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- Mobiles and calculators are not allowed in this exam.
  - Provide all necessary steps required in the solution.
  - Attempt all questions to the point.
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Question #	Marks	Maximum Marks
1		$5 + 5 = 10$
2		$4 + 3 = 7$
3		$9 + 6 = 15$
4		$4 + 4 + 4 = 12$
5		$4 + 4 + 4 + 4 = 16$
Total		60

## Full Exam paper

1. (a) Draw the hexagon inscribed in a unit circle whose vertices represent the complex numbers  $z$  such that  $z^6 = 1$ .  
(b) Evaluate  $(-\sqrt{3} - i)^{29}$ .
2. (a) Define domain and region in the complex plane.  
(b) Is the set  $3 \leq |z| < 11$  a domain or region or both? Give reason.
3. (a) Let  $f(z) = z + 1$ ,  $g(z) = e^{i\frac{\pi}{4}}z$  and  $h(z) = 2z$  then sketch the image of the unit semi-disk  $|z| \leq 1$ ,  $\text{Im}(z) > 0$  under the following maps.
  - i.  $h(z)$
  - ii.  $g(h(z))$
  - iii.  $f(g(h(z)))$(b) By using  $\epsilon - \delta$  definition of limit show that  $\lim_{z \rightarrow i}(2z + 1) = 1 + 2i$  by finding  $\delta$  in terms of  $\epsilon$ .
4. Determine the points where the following functions are differentiable and analytic.
  - (a)  $f(z) = \bar{z}$
  - (b)  $f(z) = (3x^2 + y + 3) + i(y^2 - x)$
  - (c)  $f(z) = (3x^2 + 2x - 3y^2 - 1) + i(6xy + 2y)$
5. Write down  $T$  for a true and  $F$  for a false statement by supporting your answer with appropriate reason.
  - (a) Let  $f$  be an analytic function on  $D := \{|z| < 1\} \cup \{|z| > 1\}$  such that  $f'(z) = 0$ . Then  $f$  is constant on  $D$ .
  - (b) The function  $u(x, y) = e^{-x} \sin y$  can be real part of some analytic function.
  - (c)  $f(z) = \frac{|z| + z}{2}$  is analytic at the origin.
  - (d) Let  $f(z) = 3 + iv(x, y)$  be an analytic function then  $v(x, y)$  must be constant.
  - (e) The curves  $3x^2 + 2x - 3y^2 - 1 = c_1$  and  $6xy + 2y = c_2$  are level curves for arbitrary constant  $c_1$  and  $c_2$ .