

**King Fahd University of Petroleum & Minerals**  
**Department of Mathematics & Statistics**  
**Math 445 Major Exam 2**  
**The Second Semester of 2021-2022 (212)**

**Time Allowed: 90 Minutes**

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Name: \_\_\_\_\_ ID#: \_\_\_\_\_

Section/Instructor: \_\_\_\_\_ Serial #: \_\_\_\_\_

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- Mobiles and calculators are not allowed in this exam.
  - Provide all necessary steps required in the solution.
  - Attempt all questions to the point.
  - No credit for answers without justification.
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Question #	Marks	Maximum Marks
1		
2		
3		
4		
5		
6		
Total		60

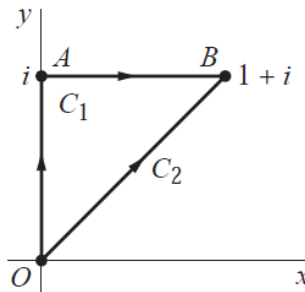
## Full Exam paper

1. (a) Find the principal value of  $\left(\frac{e}{2}(-1 + i\sqrt{3})\right)^{5\pi i}$ .
- (b) Determine and sketch the domain of analyticity of  $f(z) = \text{Log}(39 + i - z)$ . Calculate  $f'(z)$ .

2. (a) Identify the interior of the simple closed contour given in the figure. Is it positively oriented?

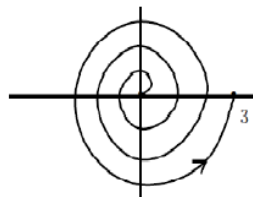


- (b) Evaluate  $\int_C f(z)dz$  where  $f(z) = y - x - i3x^2$  and  $C$  is the contour given in the following figure.



3. (a) Show that  $\left|\int_C \frac{1}{3 + 5z^3} dz\right| \leq \pi$  where  $C : e^{it}, 0 \leq t \leq 2\pi$ .

- (b) Evaluate  $\int_C (z^2 - 2)dz$  where  $C$  is the contour given in the following figure.



4. (a) State the definition of simply and multiply connected domains. Given an example of each type.
- (b) If  $\Gamma$  is upper semi-circle from  $z = 1$  to  $z = -1$  and  $z^i$  is the principal value then show that

$$\int_{\Gamma} \frac{z^i}{1-i} dz = \frac{1}{2}(1 + e^{-\pi})$$

- (c) State the Cauchy inequality theorem for Cauchy estimates.
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5. (a) Using the Cauchy integral formula, compute

$$\oint_{|z|=\sqrt{5}} \frac{dz}{(z-i)^2(z-3i)^2}$$

- (b) Let  $f(z) = \sum_{j=0}^{\infty} \frac{j^3}{3^j} z^j$  then compute

$$\int_{|z|=1} \frac{f(z) \cos z}{z^2} dz$$

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6. Write down  $T$  for a true and  $F$  for a false statement by supporting your answer with an appropriate reason.

- (a) 1 raise to any power is not always equal to 1.

- (b) Let  $f$  be an analytic function, inside and on  $|z| = 3$ . Then  $f$  can not attain its max value at  $z = \frac{5}{4} + i \frac{\sqrt{119}}{4}$ .

- (c) Let  $f(z) = \sum_{j=0}^{\infty} \frac{f^{(j)}(39\pi)}{j!} (z - 39\pi)^j$  and  $g(z) = \sum_{j=0}^{\infty} \frac{g^{(j)}(1 - i\pi)}{j!} (z - 1 + i\pi)^j$  be Taylor series expansions of  $f(z)$  and  $g(z)$ . Then it is possible that  $f(z) = g(z)$ . If the statement is true, give an example of such a function.