King Fahd University of Petroleum and Minerals
Department of Mathematics
MATH445 - Intro. to Complex Variables
Major Exam I – Semester 221

Exercise 1

1. Find all the values of the following
   (a) $1^{1/6}$
   (b) $\left(\frac{1-\sqrt{3}i}{1+i}\right)^{1/4}$
2. Solve the equation $(1+z)^6 = z^6$.

Exercise 2

Describe the projections on the Riemann sphere of the following sets in the complex plane

(a) The upper-half-plane $\{z : \operatorname{Im} z > 0\}$.

(b) The line $x + y + 1 = 0$.

Exercise 3

Show that the mapping $w = -\dfrac{1}{z}$ corresponds to a π- rotation of the Riemann sphere about the x_2-axis.

Exercise 4

Let

$$w = J(z) = z + \frac{a^2}{z}, \quad (a > 0).$$

Show that

(a) Show that J maps $|z| = a$ onto the real interval $[-2a, 2a]$.

(b) Show that J maps $|z| = b$ with $(b > a)$ onto an ellipse.

Exercise 5

1. Show that if f and $\overline{f}$ are analytic in a domain D. Show that f is constant
2. Show that if $w = f(z) = u + iv$ is analytic and maps D into a line $v = au$, then f must be a constant.

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Exercise 6

Show that $u(x, y) = x^3 - 3xy^2 + y$ is harmonic and find v a harmonic conjugate of u.

Exercise 7

If u and v are expressed in terms of polar coordinates (r, θ), show that the Cauchy-Riemann equations can be written in the form

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \quad \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

[Hint: Consider the quotient $\dfrac{f(z) - f(z_0)}{z - z_0}$ as $z \to z_0 = r_0 e^{i\theta_0}$ along the ray $\theta = \theta_0$ and along the circle $|z| = r_0$.]