EXAM 1

Duration: 120 minutes

ID:	
NAME:	

- Show your work.
- There are empty pages attached to this exam booklet.

Problem	Score
1	
2	
3	
4	
5	
Total	/100

Problem 1 (20 points)

Let $f : A \to B$ be an isomorphism between two linear vector spaces A and B. If $\mathfrak{B}$ is a Hamel basis for A, show that $f(\mathfrak{B})$ is a Hamel basis for B.

Problem 2 (20 points)

Let $\mathcal{C}[-1,1]$ denote the linear vector space of all real-valued continuous functions on the interval $[-1,1]$. Further let

$$A = \{f \in \mathcal{C}[-1,1] : f(-x) = f(x), \quad \text{for all} \quad x \in [-1,1]\},$$
$$B = \{f \in \mathcal{C}[-1,1] : f(-x) = -f(x), \quad \text{for all} \quad x \in [-1,1]\}.$$

Show that $\mathcal{C}[-1,1] = A \oplus B$.

Problem 3 (20 points)

Let A be a vector space and let $f \in A^*$ be a non-zero linear functional. Show that the null-space $\mathcal{N}(f)$ is a maximal subspace of A.

Problem 4 (20 points)

Let $\mathbb{C}$ be the set of complex numbers. Define $d : \mathbb{C}^2 \to \mathbb{R}^+$ as

$$d(w,z) = \begin{cases} |w-z| & \text{if } \arg z = \arg w \text{ or one of } z \text{ and } w \text{ is zero,} \\ |w|+|z| & \text{otherwise} \end{cases}$$

where $w, z \in \mathbb{C}$. Show that d is a metric on $\mathbb{C}$.

Problem 5 (20 points)

Let (X, d) be a metric space such that every closed ball is compact. Show that X is complete.