STAT 416- Major Exam 2

KFUPM, Department of Mathematics and Statistics

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1 Exercise 1(11=4+2+3+2 points)

1. Consider a Markov process with states 0, 1, 2 and which the following transition ratio matrix Q

$$Q = \begin{pmatrix} -\lambda & \lambda & 0 \\ \mu & -\lambda-\mu & \lambda \\ \mu & 0 & -\mu \end{pmatrix}$$

   Where $\lambda > 0$, $\mu > 0$. Derive the parameters v_i and $p_{i,j}$ for this Markov process.

2. Define an $M/M/1$ system.

3. Define a Poisson Process.

4. Write down the Backward-Kolmogorov equation for a continuous Markov chain.

2 Exercise 2(10=5+5 points)

Suppose that the amount of time one spends in a bank is exponentially distributed with mean ten minutes.

1. What is the probability that a customer will spend more than fifteen minutes in the bank?

2. What is the probability that a customer will spend more than fifteen minutes in the bank given that she is still in the bank after ten minutes?

3 Exercise 3 (7 points)

Consider a continuous Markov chain on $\{0, 1, 2, 3, 4\}$ where $q_{i,i+1} = i + 1$ for $i = 0, 1, 2, 3$, $q_{i,i-1} = 5 - i$ for $i = 1, 2, 3, 4$, and $q_{i,j} = 0$ otherwise. Find the stationary distribution vector.

4 Exercise 4(16=4+4+4+4 points)

A small barbershop, operated by a single barber, has room for at most two customers. Potential customers arrive at a Poisson rate of three per hour, and successive service time are independent exponential random variable with mean 1/4 hour.

1. Give the generator matrix of the number of customers in the shop.

2. Find the stationary distribution.

3. What is the average number of customers in the shop?

4. What is the proportion of potential customers that inter the shop?

5 Exercise 5(15=5+5+5 points)

Customers entered a supermarket are classified as men and women, they entered the supermarket according to independent Poisson processes having respective rates of two men and four women per hour.

1. What is the probability that the total number of customers entered the supermarket will be less than 2 during the first hour of working?

2. What is the probability that in one hour exactly 3 men and 3 women enter the supermarket?

3. What is the probability that 2 men will arrive before the first woman arrives?

6 Exercise 6(16=4+4+4+4 points)

Students arrive at the campus post office according to a Poisson process with an average rate of one student every 4 minutes. The time required to serve each student is exponentially distributed with a mean of 3 minutes. There is only one postal worker at the counter, and any arriving student that finds the worker busy joins a queue.

1. Specify the type of queueing system and find the corresponding parameters.

2. Show that the stationary distribution exists and find it.

3. What is the mean number of students at the postal office?

4. What is the mean number of students who are waiting at the postal office?

Notes

1.
$$\sum_{n=m}^{\infty} x^n = \frac{x^m}{1-x},$$
$$\sum_{n=m}^{\infty} n x^n = \frac{x^m(m+x-mx)}{(1-x)^2},$$

for any integer $m \geq 0$.

2.
$$\pi_n = \frac{\theta_n}{\sum_{n=0}^{\infty} \theta_n},$$

for $n = 0, 1, 2, \ldots$, where $\theta_0 = 1$ and $\theta_n = \frac{\lambda_0 \cdots \lambda_{n-1}}{\mu_1 \cdots \mu_n}$ for $n \geq 1$.