All measurements are approximations--no measuring device can give perfect measurements without experimental uncertainty. By convention, a mass measured to 13.2 g is said to have an absolute uncertainty of 0.1 g and is said to have been measured to the nearest 0.1 g. In other words, we are somewhat uncertain about that last digit—it could be a "2"; then again, it could be a "1" or a "3". A mass of 13.20 g indicates an absolute uncertainty of 0.01 g.

The objectives of this tutorial are:

—Explain the concept of significant figures.
—Define rules for deciding the number of significant figures in a measured quantity.
—Explain the concept of an exact number.
—Define rules for determining the number of significant figures in a number calculated as a result of a mathematical operation.
—Explain rules for rounding numbers.
—Present guidelines for using a calculator.
—Provide some exercises to test your skill at significant figures.

What is a "significant figure"?

The number of significant figures in a result is simply the number of figures that are known with some degree of reliability. The number 13.2 is said to have 3 significant figures. The number 13.20 is said to have 4 significant figures

**Rules for deciding the number of significant figures in a measured quantity:**

(1) All nonzero digits are significant:

1.234 g has 4 significant figures,
1.2 g has 2 significant figures.

(2) Zeroes between nonzero digits are significant:
1002 kg has 4 significant figures,
3.07 mL has 3 significant figures.

(3) Leading zeros to the left of the first nonzero digits are not significant; such zeroes merely indicate the position of the decimal point:

0.001 °C has only 1 significant figure,
0.012 g has 2 significant figures.

(4) Trailing zeroes that are also to the right of a decimal point in a number are significant:

0.0230 mL has 3 significant figures,
0.20 g has 2 significant figures.

(5) When a number ends in zeroes that are not to the right of a decimal point, the zeroes are not necessarily significant:

190 miles may be 2 or 3 significant figures,
50,600 calories may be 3, 4, or 5 significant figures.

The potential ambiguity in the last rule can be avoided by the use of standard exponential, or "scientific," notation. For example, depending on whether the number of significant figures is 3, 4, or 5, we would write 50,600 calories as:

5.06 × 10^4 calories (3 significant figures)
5.060 × 10^4 calories (4 significant figures), or
5.0600 × 10^4 calories (5 significant figures).

**What is a "exact number"?**

Some numbers are exact because they are known with complete certainty.

Most exact numbers are integers: exactly 12 inches are in a foot, there might be exactly 23 students in a class. Exact numbers are often found as conversion factors or as counts of objects.

Exact numbers can be considered to have an infinite number of significant figures. Thus, the number of apparent significant figures in any exact number can be ignored as a limiting factor in determining the number of significant figures in the result of a calculation.

**Rules for mathematical operations**

In carrying out calculations, the general rule is that the accuracy of a calculated result is limited by the least accurate measurement involved in the calculation.
In addition and subtraction, the result is rounded off to the last common digit occurring furthest to the right in all components. Another way to state this rule is, in addition and subtraction, the result is rounded off so that it has the same number of decimal places as the measurement having the fewest decimal places. For example,

\[100 \text{ (assume 3 significant figures)} + 23.643 \text{ (5 significant figures)} = 123.643,\]

which should be rounded to 124 (3 significant figures).

### Addition and Subtraction with Significant Figures

When combining measurements with different degrees of accuracy and precision, the accuracy of the final answer can be no greater than the least accurate measurement. This principle can be translated into a simple rule for addition and subtraction: When measurements are added or subtracted, the answer can contain no more decimal places than the least accurate measurement.

\[
\begin{align*}
150.0 \text{ g H}_2\text{O} \quad \text{(using significant figures)} \\
+ 0.507 \text{ g salt} \\
\hline
150.5 \text{ g solution}
\end{align*}
\]

In multiplication and division, the result should be rounded off so as to have the same number of significant figures as in the component with the least number of significant figures. For example,

\[3.0 \text{ (2 significant figures)} \times 12.60 \text{ (4 significant figures)} = 37.8000\]

which should be rounded off to 38 (2 significant figures).

### Multiplication and Division With Significant Figures

The same principle governs the use of significant figures in multiplication and division: the final result can be no more accurate than the least accurate measurement. In this case, however, we count the significant figures in each measurement, not the number of decimal places: When measurements are multiplied or divided, the answer can contain no more significant figures than the least accurate measurement.

Example: To illustrate this rule, let's calculate the cost of the copper in an old penny that is pure copper. Let's assume that the penny has a mass of 2.531 grams, that it is essentially pure copper, and that the price of copper is 67 cents per pound. We can start by from grams to pounds.
We then use the price of a pound of copper to calculate the cost of the copper metal.

\[
2.531 \text{ g} \times \frac{1 \text{ lb}}{453.6 \text{ g}} = 0.005580 \text{ lb}
\]

\[
0.005580 \text{ lb} \times \frac{67 \text{ g}}{1 \text{ lb}} = 0.3749 \text{ g}
\]

There are four significant figures in both the mass of the penny (2.531) and the number of grams in a pound (453.6). But there are only two significant figures in the price of copper, so the final answer can only have two significant figures.

**General guidelines for using calculators**

When using a calculator, if you work the entirety of a long calculation without writing down any intermediate results, you may not be able to tell if an error is made. Further, even if you realize that one has occurred, you may not be able to tell where the error is.

In a long calculation involving mixed operations, carry as many digits as possible through the entire set of calculations and then round the final result appropriately. For example,

\[
(5.00 / 1.235) + 3.000 + (6.35 / 4.0) = 4.04858... + 3.000 + 1.5875 = 8.630829...
\]

The first division should result in 3 significant figures. The last division should result in 2 significant figures. The three numbers added together should result in a number that is rounded off to the last common significant digit occurring furthest to the right; in this case, the final result should be rounded with 1 digit after the decimal. Thus, the correct rounded final result should be 8.6. This final result has been limited by the accuracy in the last division.

**Warning:** carrying all digits through to the final result before rounding is critical for many mathematical operations in statistics. Rounding intermediate results when calculating sums of squares can seriously compromise the accuracy of the result.

**Rounding Off**

When the answer to a calculation contains too many significant figures, it must be rounded off.

There are 10 digits that can occur in the last decimal place in a calculation. One way of rounding off involves *underestimating* the answer for five of these digits (0, 1, 2, 3, and
4) and *overestimating* the answer for the other five (5, 6, 7, 8, and 9). This approach to rounding off is summarized as follows.

If the digit is smaller than 5, drop this digit and leave the remaining number unchanged. Thus, 1.684 becomes 1.68.

If the digit is 5 or larger, drop this digit and add 1 to the preceding digit. Thus, 1.247 becomes 1.25.

**Sample problems on significant figures**

*Instructions:* print a copy of this page and work the problems. When you are ready to check your answers, go to the next page.

1. $37.76 + 3.907 + 226.4 = $
2. $319.15 - 32.614 = $
3. $104.630 + 27.08362 + 0.61 = $
4. $125 - 0.23 + 4.109 = $
5. $2.02 \times 2.5 = $
6. $600.0 / 5.2302 = $
7. $0.0032 \times 273 = $
8. $(5.5)^3 = $
9. $0.556 \times (40 - 32.5) = $
10. $45 \times 3.00 = $
11. $3.00 \times 10^5 - 1.5 \times 10^2 = $ (Give the exact numerical result, then express it the correct number of significant figures).
12. What is the average of 0.1707, 0.1713, 0.1720, 0.1704, and 0.1715?

**Answer key to sample problems on significant figures**

1. $37.76 + 3.907 + 226.4 = 268.1$
2. $319.15 - 32.614 = 286.54$
3. \(104.630 + 27.08362 + 0.61 = 132.32\)

4. \(125 - 0.23 + 4.109 = 129\) (assuming that 125 has 3 significant figures).

5. \(2.02 \times 2.5 = 5.0\)

6. \(600.0 \div 5.2302 = 114.7\)

7. \(0.0032 \times 273 = 0.87\)

8. \((5.5)^3 = 1.7 \times 10^2\)

9. \(0.556 \times (40 - 32.5) = 4\)

10. \(45 \times 3.00 = 1.4 \times 10^2\) (assuming that 45 has two significant figures)

11. \(3.00 \times 10^5 - 1.5 \times 10^2 = 299850 = 3.00 \times 10^5\)

12. What is the average of 0.1707, 0.1713, 0.1720, 0.1704, and 0.1715? Answer = 0.1712

**Links to other resources on the use of significant figures**

**Uncertainty in Measurements**: A tutorial by Professor Frederick A. Sensese at Frostburg State University that shows how uncertainty arises from length, temperature, and volume measurements. How to count significant figures for a single measurement and for a series of measurements. How to round measurements to the correct number of significant figures.
http://antoine.frostburg.edu/cgi-bin/senese/tutorials/sigfig/index.cgi

**Significant figures quiz**: A JavaScript tutorial by Professor Frederick A. Sensese at Frostburg State University that lets you test your knowledge of the use of significant figures.
http://antoine.frostburg.edu/chem/senese/101/measurement/sigfig-quiz.shtml

**Frequently asked questions** about measurements, including FAQs on significant figures (such as: "Why should the rules for propagating significant digits not be applied to averages?" "Why does 1101 cm - 1091 cm = 10 cm with 2 significant figures?" "Are there simpler rules for counting significant digits?")
http://antoine.frostburg.edu/chem/senese/101/measurement/faq.shtml
Determining the number of significant figures: a drill by Scott Van Bramer of Widener University involving significant figures that presents you with successive number displays and grades you on your answer to the question, "How many significant figures are there?"
http://science.widener.edu/svb/tutorial/sigfigures.html

Significant Figures and Rounding Rules: a discussion of issues relating to the proper teaching of significant figures and rounding rules. This site, authored by Christopher Mulliss, includes links to other pages and to interactive tutorials.
http://www.angelfire.com/oh/cmulliss