Q1.

Two point charges \( q_1 = +5.0 \, \mu\text{C} \) and \( q_2 = -5.0 \, \mu\text{C} \) are placed on the \( x \) axis, as shown in FIGURE 1. What is the net electrostatic force on charge \( Q \) due to \( q_1 \) and \( q_2 \)? Take \( Q = +2.0 \, \mu\text{C} \).

\[
\begin{align*}
   A) & \quad +4.3 \, \hat{i} \,(\text{mN}) \\
   B) & \quad -4.3 \, \hat{i} \,(\text{mN}) \\
   C) & \quad +4.3 \, j\,(\text{mN}) \\
   D) & \quad -4.3 \, j\,(\text{mN}) \\
   E) & \quad \text{zero}
\end{align*}
\]

Ans:

\( \vec{F}_1 \) and \( \vec{F}_2 \) have the same magnitude and their \( y \) – components cancel.

\[
\therefore \quad \vec{F}_Q = 2. \vec{F}_1 \cdot \cos \theta \, \hat{i}
\]

\[
\begin{align*}
   & = 2 \times \frac{kq_1Q}{r^2} \cdot \frac{3}{r} = \frac{6kq_1Q}{r^3} \, \hat{i} \\
   & = 6 \times 9 \times 10^9 \times 5 \times 2 \times 10^{-12} \\
   & = 4.3 \times 10^{-3} \, \hat{i} \,(\text{N}) \\
   & = 4.3 \, \hat{i} \,(\text{mN})
\end{align*}
\]

Q2.

Two identical conducting spheres carry charges of \(+5.0 \, \mu\text{C}\) and \( -1.0 \, \mu\text{C} \). The centers of the spheres are initially separated by a distance \( L \). The two spheres are brought together so that they are in contact. The spheres are then returned to their original separation \( L \). What is the ratio of the magnitude of the electric force on either sphere after the spheres are touched to that before they were touched?

\[
\begin{align*}
   \text{A) } & \quad \frac{4}{5} \\
   \text{B) } & \quad \frac{1}{5} \\
   \text{C) } & \quad \frac{9}{5} \\
   \text{D) } & \quad \frac{5}{1} \\
   \text{E) } & \quad \frac{4}{9}
\end{align*}
\]

Ans:

Before: \( F_1 = \frac{(k)(5)(1)}{L^2} \)

After: Charge on each is: \( \frac{+5 - 1}{2} = +2\mu\text{C} \) \( \Rightarrow \) \( F_2 = \frac{(k)(2)(2)}{L^2} \)

\[
\begin{align*}
   \frac{F_2}{F_1} & = \frac{4k \cdot L^2}{L^2 \cdot 5k} = \frac{4}{5}
\end{align*}
\]
Q3.

A uniform electric field, with a magnitude of 4 N/C, points in the positive $x$ direction. When a charge is placed at the origin, the resulting electric field on the $x$ axis at $x = 2$ m becomes zero. With the charge still at the origin, what is the magnitude of the electric field at $x = +4$ m?

A) 3 N/C  
B) 2 N/C  
C) 1 N/C  
D) 4 N/C  
E) 0 N/C

Ans:
The charge must be negative.
Its magnitude is: $q = \frac{E r^2}{k} = \frac{4 \times 4}{9 \times 10^4} = \frac{16}{9}$ NC

Now, consider $x = +4$m:
$E_{\text{net}} = E_0 + E_q = 4\hat{i} - \left(9 \times 10^9 \frac{16}{9} \times 10^{-9}\right)\hat{i} = 3\hat{i}$ (N/C)

Q4.

An electron, initially moving with a velocity of $3.0 \times 10^4 \hat{i}$ (m/s), enters a region of a uniform electric field that is parallel to the $x$ axis. The electron comes to rest after travelling a distance of 2.5 cm in the field. What is the electric field? Ignore gravity.

A) $+0.10\ \hat{i}$ (N/C)  
B) $-0.10\ \hat{i}$ (N/C)  
C) $+9.8\ \hat{i}$ (N/C)  
D) $-9.8\ \hat{i}$ (N/C)  
E) $+1.9\ \hat{i}$ (N/C)

Ans:

The electric field must be in the (+) $x$ direction.

$\vec{v}_f^2 = v_i^2 - 2ax \Rightarrow a = \frac{v_i^2}{2x}$

$F = ma \Rightarrow qE = ma \Rightarrow E = \frac{ma}{q}$

$E = \frac{m}{e} \cdot \frac{v_i^2}{2x} = \frac{9.11 \times 10^{-31}}{1.6 \times 10^{-19}} \times \frac{9 \times 10^8}{5 \times 10^{-2}} = 0.10$ N/C
Q5.
An electric dipole is placed in a 300 N/C uniform electric field with its dipole moment initially perpendicular to the field. The dipole rotates until its dipole moment makes an angle of 10° with the electric field. If the dipole moment has a magnitude of $2 \times 10^{-9}$ C.m, the work done by the electric field is

A) $+6 \times 10^{-7}$ J  
B) $-6 \times 10^{-7}$ J  
C) 0  
D) $+1 \times 10^{-7}$ J  
E) $-1 \times 10^{-7}$ J

Ans:
\[
U_i = -\vec{p} \times \vec{E} = -p. E. \cos \theta_i = 0
\]
\[
U_f = -p. E. \cos \theta_f = -2 \times 10^{-4} \times 300 \times \cos 10^\circ = -5.9 \times 10^{-7} \text{ J}
\]
\[
W = -\Delta U = -(U_f - U_i) = -U_f = +5.9 \times 10^{-7} \text{ J}
\]
\[
\Rightarrow + 6.0 \times 10^{-7} \text{ J}
\]

Q6.
A hemisphere has a charge of $+6.6 \times 10^{-7}$ C enclosed inside it. It is placed in a uniform electric field, as shown in cross section in FIGURE 2. The electric flux through the curved portion of the hemisphere is $+9.8 \times 10^4$ N.m²/C. What is the electric flux through the flat base of the hemisphere?

A) $-2.3 \times 10^4$ N.m²/C  
B) $+2.3 \times 10^4$ N.m²/C  
C) $-9.8 \times 10^4$ N.m²/C  
D) $+9.8 \times 10^4$ N.m²/C  
E) $+3.2 \times 10^4$ N.m²/C

Ans:
\[
\Phi_{\text{net}} = \Phi_{\text{curve}} + \Phi_{\text{base}}
\]
\[
\Rightarrow \Phi_{\text{base}} = \Phi_{\text{net}} - \Phi_{\text{curve}}
\]
\[
\frac{Q_{\text{enc}}}{\varepsilon_0} = \Phi_{\text{curve}}
\]
\[
= + \frac{6.6 \times 10^{-7}}{8.85 \times 10^{-12}} - 9.8 \times 10^4 = -2.3 \times 10^4 \text{ (N.m²/C)}
\]
Q7.
A +6.0-nC point charge is placed at the center of a hollow spherical conductor (inner radius = 1.0 cm, outer radius = 2.0 cm) which has a net charge of +4.0 nC. Determine the resulting charge density on the inner surface of the sphere.

A) $-4.8 \mu C/m^2$
B) $+4.8 \mu C/m^2$
C) $-9.5 \mu C/m^2$
D) $+9.5 \mu C/m^2$
E) $-8.0 \mu C/m^2$

Ans:

$q_{\text{inner}} = -6.0 \text{ nC}$

$\sigma_{\text{inner}} = \frac{q_{\text{inner}}}{A_{\text{inner}}} = \frac{q_{\text{inner}}}{4\pi R_i^2} = \frac{-6.0 \times 10^{-9}}{4\pi \times 1.0 \times 10^{-4}}$

$= -4.77 \times 10^{-6} \text{ } \frac{C}{m^2} = -4.8 \mu C/m^2$

Q8.
An infinitely long cylinder of radius $R = 2.0 \text{ cm}$ has a uniform charge density $\rho = 18 \mu C/m^3$. Calculate the magnitude of the electric field at a distance of $r = 4.0 \text{ cm}$ from the axis of the cylinder.

A) $1.0 \times 10^4 \text{ N/C}$
B) $2.5 \times 10^3 \text{ N/C}$
C) $5.1 \times 10^3 \text{ N/C}$
D) $2.0 \times 10^3 \text{ N/C}$
E) 0

Ans:

The point is outside the cylinder

$\int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$

$E \left(2\pi rh\right) = \frac{\rho}{\varepsilon_0} \cdot \pi R^2 h$

$\Rightarrow E = \left(\frac{\rho R^2}{2\varepsilon_0}\right) \cdot \frac{1}{r} = \frac{1.8 \times 10^{-6} \times 4.0 \times 10^{-4}}{2 \times 8.85 \times 10^{-12} \times 0.04} = 1.0 \times 10^4 \text{ N/C}$
Q9.
Two thin, infinite, charged non-conducting parallel sheets are separated by a distance \( d \). The charge density on one sheet is \(+\sigma\), and the charge density on the other sheet is \(+2\sigma\). The electric field in the region between the sheets has a magnitude of

\[
A) \frac{\sigma}{2\varepsilon_0} \\
B) \frac{\sigma}{\varepsilon_0} \\
C) \frac{3\sigma}{\varepsilon_0} \\
D) 0 \\
E) \frac{3\sigma}{2\varepsilon_0}
\]

Ans:
Between the sheets:
\[
E_{\text{net}} = E_2 - E_1 = \frac{2\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = \frac{2\sigma}{2\varepsilon_0}
\]

Q10.
A uniformly charged solid insulating sphere has radius \( R \). The electric field at \( r = 3R \) has a value of 150 N/C. What is the magnitude of the electric field at \( r = \frac{R}{3} \)? [\( r \) is the distance from the center of the sphere]

\[
A) 450 \text{ N/C} \\
B) 50.0 \text{ N/C} \\
C) 675 \text{ N/C} \\
D) 225 \text{ N/C} \\
E) 375 \text{ N/C}
\]

Ans:
Outside: \( E_1 = \frac{kQ}{r^2} = \frac{kQ}{(3R)^2} = \frac{kQ}{9R^2} \)

Inside: \( E_2 = \frac{kQ}{R^3} \cdot r = \frac{kQ \cdot R}{R^3 \cdot 3} = \frac{kQ}{3R^2} \)

\[
\Rightarrow \frac{E_2}{E_1} = \frac{kQ}{3R^2} \cdot \frac{9R^2}{kQ} = 3 \Rightarrow E_2 = 3 E_1 = 3 \times 150 = 450 \text{ N/C}
\]
Q11. Suppose that a region of space has a uniform electric field, directed towards the right, as shown in FIGURE 3. Which one of the following statements is CORRECT?

\[
\begin{align*}
\text{Fig# 3} \\
&\quad \quad \uparrow \\
&\quad \quad \quad \quad E \\
&\quad \quad \downarrow \\
&\quad \quad \quad \quad A \\
&\quad \quad \quad \quad C \\
&\quad \quad \downarrow \\
&\quad \quad \quad \quad B \\
\end{align*}
\]

A) \( V_A = V_B \) and \( V_A > V_C \)  
B) \( V_A = V_B = V_C \)  
C) \( V_A = V_B \) and \( V_A < V_C \)  
D) \( V_A > V_B > V_C \)  
E) None of the other choices

Ans: Electric field lines point toward regions of lower electric potential.

Q12. Two point charges \( Q_A = +2 \, \mu\text{C} \) and \( Q_B = -6 \, \mu\text{C} \) are located on the x-axis at \( x_A = -1 \, \text{cm} \) and \( x_B = +2 \, \text{cm} \). Where should a third charge, \( Q_C = +3 \, \mu\text{C} \), be placed on the positive x-axis so that the net electric potential at the origin is equal to zero?

\[
\begin{align*}
\text{A) } x &= 3 \, \text{cm} \\
\text{B) } x &= 1 \, \text{cm} \\
\text{C) } x &= 5 \, \text{cm} \\
\text{D) } x &= 4 \, \text{cm} \\
\text{E) } x &= 6 \, \text{cm}
\end{align*}
\]

Ans:

\[
V_0 = k \left[ \frac{q_A}{0.01} + \frac{q_B}{0.02} + \frac{q_C}{x} \right]
\]

if \( V_0 = 0 \):

\[
\frac{q_C}{x} = - \left( \frac{q_A}{0.01} + \frac{q_B}{0.02} \right)
\]

\[
= - \left( \frac{2 \times 10^{-6}}{0.01} - \frac{6 \times 10^{-6}}{0.02} \right) = 1 \times 10^{-4}
\]

\( \Rightarrow x = 0.03 \, \text{m} = 3 \, \text{cm} \)
Q13.
Over a certain region of space, the electric potential is given by \( V = (5.0 \, x) - (3.0 \, x^2y) \), where \( V \) is in volts, and \( x \) and \( y \) are in meters. What is the electric field at the point (1.0, 1.0) m?

A) \(+1.0 \, \hat{i} + 3.0 \, \hat{j} \, (V/m)\)
B) \(+1.0 \, \hat{i} - 3.0 \, \hat{j} \, (V/m)\)
C) \(-1.0 \, \hat{i} + 3.0 \, \hat{j} \, (V/m)\)
D) \(-1.0 \, \hat{i} - 3.0 \, \hat{j} \, (V/m)\)
E) \(+5.0 \, \hat{i} + 3.0 \, \hat{j} \, (V/m)\)

Ans:
\[
E_x = -\frac{\partial V}{\partial x} = -(5 - 6xy) = 6xy - 5
\]
\[
E_y = -\frac{\partial V}{\partial y} = -(-3x^2) = 3x^2
\]

at the point:
\[
E_x = (6 \times 1 \times 1) - 5 = 1.0 \, V/m
\]
\[
E_y = 3 \times 1 = 3.0 \, V/m
\]
\[\Rightarrow \vec{E} = +1.0 \, \hat{i} + 3.0 \, \hat{j} \, (V/m)\]

Q14.
Four identical point charges, each with charge \( q = +30 \mu C \), are placed at the corners of a rectangle, as shown in FIGURE 4. How much work must be done by an external agent to bring a charge \( Q = +56 \mu C \) from infinity to point P, located at the midpoint of one of the 6.0-m long sides of the rectangle?

A) +16 J
B) +22 J
C) −22 J
D) +19 J
E) −16 J

Ans:
Note that \( r = 5 \, m \)
\[
V_p = k \left[\frac{2q}{d} + \frac{2q}{r}\right] = 2kq \left[\frac{1}{r} + \frac{1}{d}\right]
\]
\[
= 2 \times 9 \times 10^9 \times 3 \times 10^{-5} \times \left(\frac{1}{5} + \frac{1}{3}\right) = 2.88 \times 10^5 \, V
\]
\[\Rightarrow W_Q = Q \cdot V_p = 56 \times 10^{-6} \times 2.88 \times 10^5
\]
\[= 15.84 \, J \rightarrow 16 \, J\]
Q15.  
A particle of charge $+2.0 \times 10^{-3}$ C moves in a region where only electric forces act on it. The particle has a kinetic energy of 5.0 J at point A. The particle then passes through point B which has an electric potential of +1.5 kV relative to point A. Determine the kinetic energy of the particle as it passes through point B.

A) 2.0 J  
B) 3.0 J  
C) 5.0 J  
D) 8.0 J  
E) 10 J

Ans:

\[ K_A + U_A = K_B + U_B \]
\[ K_B = K_A + (U_A - U_B) = K_A + q \cdot \Delta V_{BA} \]
\[ = (5.0) + (2.0 \times 10^{-3})(-1.5 \times 10^3) = 2.0 \text{ J} \]

Q16.  
Two concentric conducting spherical shells are shown in FIGURE 5. The outer sphere has charge $q_B = -5.0 \mu C$ and radius $R_B = 5.0$ m. The inner sphere has charge $q_A = +2.0 \mu C$ and radius $R_A = 2.0$ m. What is the electric potential at $r = 3.0$ m, where $r$ is the distance from the center. Take $V = 0$ at infinity.

A) $-3.0$ kV  
B) $+3.0$ kV  
C) $+15$ kV  
D) $-15$ kV  
E) zero

Ans:

\[ V_P = V_B + V_A \]
\[ = \frac{kq_B}{R_B} + \frac{kq_A}{r} \]
\[ = -\frac{9 \times 10^9 \times 5 \times 10^{-6}}{5} + \frac{9 \times 10^9 \times 2 \times 10^{-6}}{3} \]
\[ -9 \times 10^3 + 6 \times 10^3 = -3 \times 10^3 \text{ V} \]
\[ = -3 \text{ kV} \]
Q17. Two parallel conducting plates that are initially uncharged are separated by 1.2 mm. What charge must be transferred from one plate to the other if 12 kJ of energy are to be stored in the region between the plates? The area of each plate is 19 mm².

A) 58 μC  
B) 39 μC  
C) 78 μC  
D) 35 μC  
E) 93 μC

Ans:
\[ C = \frac{\varepsilon_0 A}{d} \]
\[ U = \frac{1}{2} CV^2 = \frac{q^2}{2C} \Rightarrow q^2 = 2CU = \frac{2\varepsilon_0 AU}{d} \]

\[ q = \sqrt{\frac{2 \times 8.85 \times 10^{-12} \times 19 \times 10^{-6} \times 12 \times 10^3}{1.2 \times 10^{-3}}} \]

\[ = 5.8 \times 10^{-5} \text{ C} = 58 \mu\text{C} \]

Q18. Three capacitors are arranged as shown in FIGURE 6. \( C_1 \) has a capacitance of 5.0 pF, \( C_2 \) has a capacitance of 10 pF, and \( C_3 \) has a capacitance of 15 pF. Find the potential difference between points A and B if the potential difference across \( C_2 \) is 30 V.

A) 180 V  
B) 55 V  
C) 60 V  
D) 50 V  
E) 82 V

Ans:
\[ V_2 = V_3 = 30 \text{ V (Parallel)} \]
\[ C_{23} = C_2 + C_3 = 10 + 15 = 25 \text{ pF (Parallel)} \]
\[ Q_{23} = C_{23} \times V_2 = 25 \times 10^{-12} \times 30 = 7.5 \times 10^{-10}\text{C} \]
\[ Q_1 = Q_{23} = 7.5 \times 10^{-10}\text{C (Series)} \]

\[ \Rightarrow V_1 = \frac{Q_1}{C_1} = \frac{7.5 \times 10^{-10}}{5.0 \times 10^{-12}} = 150 \text{ V} \]

\[ \Rightarrow V_{AB} = V_1 + V_{23} = 150 + 30 = 180 \text{ V} \]
Q19.
A 5.0-nF capacitor is charged using a 12-V battery. The battery is removed and the capacitor is connected to an identical uncharged capacitor. What then is the energy stored by the two capacitors?

A) 0.18 µJ
B) 0.36 µJ
C) 0.24 µJ
D) 0.54 µJ
E) 0.72 µJ

Ans:
Initial: \( Q_i = C \cdot V_i = 5 \times 10^{-9} \times 12 = 6 \times 10^{-8} \) C

Final: \( q_{1f} = C \cdot V_f \) \( Q_f = 2CV_f \)

\[ q_{2f} = C \cdot V_f \] \( Q_i = 2CV_f \Rightarrow V_f = \frac{Q_i}{2C} \)

\[ V_f = \frac{1}{2} \times 6 \times 10^{-8} \times 5 \times 10^{-9} = 6.0 \) V

\( C_{eq} = 2C = 10 \times 10^{-9} \) F \( U = \frac{1}{2}C_{eq}V_f^2 = 1.8 \times 10^{-7} \) J

Q20.
An air-filled parallel plate capacitor is connected to a battery and allowed to charge. Now, a slab of dielectric material is inserted completely filling the space between the plates of the capacitor while the capacitor is still connected to the battery. After inserting the dielectric

A) the charge on the capacitor increases
B) the energy stored in the capacitor decreases
C) the energy density decreases
D) the voltage across the capacitor increases
E) the magnitude of the electric field increases

Ans:
A